PERSPECTIVES ON EXPANSIONS

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SETTING

M is a structure for a language L, A is a subset of M.

$L^* = L(P)$ is the expansion of L by one unary predicate and (M, A) is the L^* structure where P is interpreted by A.

When does (M, A) have the same stability class as M?

TWO FACTORS

What structure does M ‘induce’ on A?

How does A ‘sit in’ M?

INDUCE

The basic formulas induced on A can be:

L^*: the traces on A of parameter free L-formulas (*induced structure*);

$L^{\#}$: the traces on A of parameter free $L(P)$-formulas ($L^{\#}$-*induced structure*, $A^{\#}$);

If M is stable, for L^*, we may allow parameters from M but not for $L^{\#}$.

EXAMPLES

Form a structure M with a two sorted universe:

1. The complex numbers.

2. A fibering over the complex numbers.

Let N extend M by putting one new point in the fiber over a if and only if a is a real number.

Now M and N are isomorphic and are ω-stable nfcp. But the structure (N, M) is unstable.

The $*$-induced structure on M is stable since in fact no new sets are definable.

In the $\#$-induced structure

$$(\exists x)E(x,y) \wedge x \notin P$$

defines the reals so the $\#$-induced structure is unstable.

SITS

Definition 1 *M is ω-saturated over A, (A is small in M), if for every $\overline{a} \in M - A$, every L-type $p \in S(\overline{a}A)$ is realized in M.*

Definition 2 *1. The set A is* weakly benign *in M if for every $\alpha, \beta \in M$ if:*

$$\mathsf{stp}(\alpha/A) = \mathsf{stp}(\beta/A)$$

implies

$$\mathsf{tp}_*(\alpha/A) = \mathsf{tp}_*(\beta/A).$$

2. (M, A) is *uniformly weakly benign* if every (N, B) which is $L(P)$-elementarily equivalent to (M, A) is weakly benign.

SUFFICIENT CONDITIONS

Explaining Baldwin-Benedikt,

Casanovas-Ziegler prove:

Theorem 3 *If* (M, A) *has the nfcp (over* A*), is small, and the* $*$*-induced theory on* A *is stable then* (M, A) *is stable.*

Extending Casanovas-Ziegler,

Baizhanov-Baldwin prove:

Theorem 4 *If* (M, A) *is uniformly weakly benign and the #-induced theory on* A *is stable then* (M, A) *is stable.*

Baizhanov, Baldwin, Shelah showed:

Theorem 5 *If* M *is superstable* (M, A) *is uniformly weakly benign for any* A.

Question 6 *If M is stable must (M, A) is uniformly weakly benign for any A?*

Question 7 *Is there a stable structure M and an infinite set of indiscernibles I such I is not indiscernible in (M, I)?*