Book Corrections

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I have listed both harmless typos and items that may cause difficulty. The serious items are marked with *.

x_1 These deduction of ω-stablity from $\aleph_1$ categoricity in Theorem 18.16 uses Keisler's result but also relies essentially on two results of Shelah, 17.11 and 6.3.1.

9^5 : Insert 'formulas' before 'with'.

9 : The assumption of Lemma 2.12 should allow G to be empty or a countable member of $\boldsymbol{K}$.

11 : Proof of 2.19. In second line should be 'finite subsets of X' not 'of $\mathcal{A}$'.

12^9 : $\psi \restriction X$ for f

12^{10}: X for X_k; Z for Z_k

12^{12}: X for X_k; it might have been clearer to write $\hat{\psi}_k$ rather than $\hat{\psi}$

18^1: 'an' for 'and'

18_5: $b_2^{\frac{1}{2}}$ for $b_2^{1\frac{1}{2}}$

19_{19}: $q_\ell h_\ell$ for $h_\ell b_\ell$

20^{19} : Delete 'from'.

20^{22} : The mysterious subscript frac1m should read $\frac{1}{m}$.

21 : * Delete $G^- =$ in second line of statement of 3.12.

24 : last line of Definition 4.27: $PC\Gamma(\lambda, \lambda)$ should replace $PC(\lambda, \lambda)$.

27 : The third paragraph should contain the following. The earliest reference I have found to the notion now denoted AEC is on pages 754/755 of the Makowski's chapter XX in *Model Theoretic Logics*, cited as Mak85 in this book.

28^{18} : There is a missing colon after the second f.

41 : The reference to elementary submodel in the proof of Theorem 5.1.8 should be to L^*-elementary submodel.

47^8 α should be β in the paragraph beginning 'For successor'.

34 : Example 4.29. The example should further require that $|U(M)| = |M/U(M)|$ to ensure categoricity in the β_α. (Thanks to Tori Norquez.)

50: Example 6.1.13. The example asked for has been found by Baldwin, Hyttinen, Kesala. `http://homepages.math.uic.edu/~jbaldwin/pub/BaldwinpaperPart2.pdf`

53 : Theorem 6.3.2 hold also for $PC\Gamma(\aleph_0, \aleph_0)$ classes. This was implicit in [She87] and is explicit in [BLS].

55_{10} The analysis in Part 4 is only for $L_{\omega_1,\omega}$.

55_8 The 'unpublished' works of Shelah appear in [She09].

75_4 : 'N' for 'n'.

76_1 * In Definition 10.6 a, a chain should be required to be continuous.

87 : Lemma 11.14. Will Boney has pointed out that the hypothesis 'transitive linear order' should be replaced 'fieldable linear order'; the desired ordering is easily obtained.

93 : *** Exercise 12.9 is **Not proved**. The intended argument does not work. The claim remains open.

97^{12} : Insert after $p \in \mathrm{ga} - \mathrm{S(M)}$: 'is minimal'.

97 : This is a piece of the 'big picture' that I should have put in to discuss Theorem 12.23. A natural approach to proving Theorem 12.23 would unthinkingly assume compactness of galois types (in the sense of Definition 11.4). But this is not necessary. There is an important distinction between an assertion of compactness: every increasing chain of Galois types has an upper bound and the existence of a bound for a coherent sequence. In applications like 12.23, the coherent sequence, both the types and the witnesses are built simultaneously. Thus there is a weaker demand than to find a sequence of witness for an already given increasing family of types.

page 108: The proof of Theorem 14.8 is badly garbled and misses one step.

108_3 delete 'and $q' = q\restriction N$'.

108_2 Replace with "(1) there is a K-embedding $\alpha_n : EM_\tau(n, \Phi) \to M$ such that $\alpha_n(P^{EM_\tau(n,\Phi)}) \prec_{\boldsymbol{K}} N^1 = P(M)$; set $N_n = P^{EM_\tau(n,\Phi)}$."

page 109: Add at beginning of item 7. Define $q'_n \in N_n$ as $\alpha_n^{-1}(q\restriction\alpha_n(N_n))$. Then, $q'_{n+1} \supseteq \alpha_{n+1}(q'_n)$, so $q' = \bigcup q'_n$ is a type over $N_\omega = \bigcup_n N_n$.

last paragraph of the proof

- The first line of the proof should include 'taking M here for each of the M_α in A.3.2.b and N_1 here for the $P(M_\alpha)$.
- In the 3/4 line of the paragraph delete the parenthetical remark beginning '(The cardinality of $Q(M)$'.

In line -2 of the proof, the argument that the q'_n are increasing requires an adjustment in theorem A.3:

In A.3, Add an additional constant symbol which denotes an element in an M^* extending all the M_α and require that the indiscernibles I in A.3.2b) are indiscernible over $P(M) \cup \{d\}$. See `http://www.math.harvard.edu/~wboney/notes/SOTTNotes.pdf` for the argument for this extension a slightly different proof of 14.8.

In the case at hand we extend the given M to an M^+ where q is realized by d. By the extension to A.3 we get that the J constructed in the current argument is indiscernible over $P(M^*)d$.

Replace $EM(\boldsymbol{a}, \Phi) \approx_{P(M^*)} EM(n, \Phi)$ by $EM(\boldsymbol{a}, \Phi) \approx_{P(M^*)d} EM(n, \Phi)$.

Now the maps α_n guarantee that the types q'_n are increasing since for any $\boldsymbol{a}$ from J^n, $\boldsymbol{a}$ and $\alpha_{n+1}^{-1}(\alpha_n(\boldsymbol{a}))$ realize the same type over $P(M^*)d$. Thus $q' = \bigcup_n q'_n$ is a type over N_ω.

Replace the last two sentences of the proof by: So if c realizes q', $\alpha_n(c)$ realizes $q \restriction \mathrm{cl}_{\tau^+}(\alpha_n(\boldsymbol{a})) \cap N^1)$ contradicting clause 3d).

I thank Will Boney for pointing out the difficulties in this argument and suggesting the resolution by modifying A3.

110^{10} : Delete the second 'chapter'.

110_{14} : * The reference should be to Lemma 10.11. This incorrect reference is repeated three times on page 111.

110 : * Theorem 14.12, ρ should be assumed to be less than λ^+.

113 Definition 15.1 $p_\beta \in M_\beta$ for $p_\omega \in M_\omega$. Note that Definition 10.6 of (μ, δ)-chain requires that M_{i+1} is universal over M_i. so $\kappa(\boldsymbol{K}, \mu)$ bounds splitting chains over limit models.

116 Definition 15.9: Add unless $i_0 = j_0$ and $i_1 = j_1$.

129_{13} : * 'at least' for 'above'

129_{13} : h_{XY} for h

140_7: The initial sentence, 'The assumption that $2^{\aleph_0} < 2^{\aleph_1}$ is essential in 18.16' should be deleted. The remainder of the paragraph gives a more nuanced account of the actual situation.

141 : Exercise 18.19 should assume $A \subseteq B$.

146^{14}: '2b' should be '3b'.

146^{8}: Insert 'a' before 'bit'.

146^{9}: The example is correct but needs more explanation: Note that $\mathrm{tp}(a/B)$ does not split over C because only one new class has been added.

146^{22} : 19.14.2 - space after 'If'.

146_9: MBA' for NBA'.

147 :* The proof of Lemma 19.16 is (at best) needlessly complicated. A simpler argument like that for 19.7 suffices. Use 19.8 to find an appropriate C.

147 :* Theorem 19.18 is wrong as stated. The current statement should conclude with the phrase 'and such that p and $\hat{p}$ have the same rank.

A second clause can add. If A is Tarski-Vaught in B, then the original conclusion of 19.18 holds.

(It is easy to construct $p \in S_{\mathrm{at}}(A)$ that are stationary with A a finite set that have unique extensions to a larger set in each of various finite ranks and none of these extensions split. E.g. Let E be an equivalence relation with two classes, each infinite. Let E_2 be an equivalence relation on one of the classes and undefined otherwise. Call the prime model of this theory N.

Fix $m \in N$ with $(\exists x)E_2(x, m)$. Now $R_N(x = x) = R_N(E_1(x, m) \wedge \neg E_2(x, m)) = 4$. But then if q is the generic type of the E_1-class of m. $R_N(q)$ is also 4. So $x = x$ is stationary. But it can also be extended to a type over N with rank 2.)

The proof given is wrong-headed. The correct idea is to add a formula $\phi(\mathbf{x}, \mathbf{c})$ to p just if $R(p) = R(p \cup \{\phi(\mathbf{x}, \mathbf{c})\})$. See pages 224-225 of citeSh87a for details.

The main result has nothing to do with Lemma 19.7; Lemma 19.7 does justify the second clause. I thank Martin Bays for pointing out the error.

165_{14}: $\mathrm{cf}(\delta)$ for $\mathrm{cf}(\lambda)$.

165_5: $\mathbf{c}_2 \downarrow_{D_{i+1}} M_i D_{i+1} x$ for $\mathbf{c}_2 \downarrow_{D_{j+1}} M_i D_{i+1} x$

185 : Definition 25.1. Replace 'that' by 'where A'.

195 : In the third paragraph, the following sentence was omitted. The set $G*$ is a set of affine copies of G indexed by K.

204_{14} : 'tameness' should be 'non-tameness'.

205^{15} : Fact A2. 1) should read: $\beth_n(\kappa)^+ \to (\kappa^+)^{n+1}_\kappa$.

208^{11}. 'Now we turn to the main construction'. The induction should have length ω.

209_2 : Insert 'among' between 'are' and 'those'.

212 : Theorem B.6. The theorem should contain the hypothesis that ψ has a model that is ω_1-like. See chapter 6 of `http://homepages.math.uic.edu/~marker/math512-F13/inf.pdf` for an account which avoids this oversight.

217_{10} : Insert 'in' for 'is'.

217_9 : Insert 'to' before 'emphasize'.

222_{10} : Insert ',' after 'Shelah'.

bibliography: Shelah's book was published just after this one went to press [She09]

References

[BLS] J.T. Baldwin, Paul Larson, and S. Shelah. Saturated models of almost galois ω-stable classes. preprint -Sh index f1179 `http://homepages.math.uic.edu/~jbaldwin/pub/BlLrSh1003jul9.pdf`.

[She87] Saharon Shelah. Classification of nonelementary classes II, abstract elementary classes. In J.T. Baldwin, editor, *Classification theory (Chicago, IL, 1985)*, pages 419–497. Springer, Berlin, 1987. paper 88: Proceedings of the USA–Israel Conference on Classification Theory, Chicago, December 1985; volume 1292 of *Lecture Notes in Mathematics*.

[She09] S. Shelah. *Classification Theory for Abstract Elementary Classes*. Studies in Logic. College Publications `www.collegepublications.co.uk`, 2009. Binds together papers 88r, 600, 705, 734 with introduction E53.

[Van13] Monica VanDieren. Erratum to "categoricity in abstract elementary classes with no maximal models" [ann. pure appl. logic 141 (2006) 108-147]. *Annals of Pure and Applied Logic*, 164:131–133, 2013.