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Bratteli Diagrams for Weak Solenoids

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Questions/Problems

- We study dynamical systems given by group actions on Cantor sets. We aim to develop invariants for these systems which reflect the structure of the group and the dynamics of the action.

Theorems

- Theorem: A minimal equicontinuous action of a finitely generated group on a Cantor set has an "Almost Finite Presentation."
- Theorem (work in progress): For Γ a finitely generated group, acting minimally and equicontinuously on a Cantor set X, there exists a Bratteli-type model that represents the group action dynamics.

Bratteli Diagrams

- A *Bratteli diagram* $B = (V, E)$ is an infinite directed graph that encodes the topological information of a system.
- In the $\mathbb{Z}$ and $\mathbb{Z}^n$ case, an order is added to encode the dynamics in an ordered Bratteli diagram.
- We want to extend our model in a similar way for non-abelian groups.

Weak Solenoids

- A weak solenoid is the inverse limit of finite-to-one covering spaces of a compact manifold.
- It can be represented by a nested sequence of fundamental groups.
- The inverse limit space is homeomorphic to a generalized lamination.

Group Chains and Normalizers

- Consider an infinite chain of nested finite index proper subgroups

$$\Gamma = \Gamma_0 > \Gamma_1 > \Gamma_2 > \ldots \quad , \quad \Gamma_* = \bigcap \Gamma_i$$

- Let $X_i = \Gamma/\Gamma_i$, which is a finite non-trivial set.
- Then the inverse limit $X = X_\infty = \varprojlim X_i$ is a Cantor set.
- Γ acts on X on the left.
- Let $N_i = N(\Gamma_i)$ be the normalizer of Γ_i in Γ.
- Then N_i/Γ_i acts on X_i on the right, and commutes with the Γ-action on the left.
- Define $N_* = \bigcap N_i$.
- In general, $N_* \neq N(\Gamma_*)$.
- If every subgroup is normal, and thus every normalizer is the whole group, we call the chain *normal*.
- If the chain stabilizes, i.e. $\exists k$ such that $\forall i > k$, $N_i = N_{i+1}$, and N_* is nontrivial, then we call the chain *subnormal*.
- $N_* \subset N_i \ \forall i$, so $\lim_\leftarrow N_*/\Gamma_i$ acts on each X_i on the right, and so on X.
- Thus the left Γ-action commutes with the right N_*/Γ_i-action.

Schori Example

- The Schori Solenoid is a weak solenoid given by 3-1 covering maps with the base space X_0 a surface of genus 2.
- For $\Gamma_i = \pi_i(X_i)$, $N(\Gamma_i) = \Gamma_i$, so the chain of normalizers is strictly decreasing and never stabilizes. Thus, the chain is neither normal nor subnormal.

Heisenberg Examples

- The discrete Heisenberg group H is the set $\mathbb{Z}^3$ with the group operation $(x, y, z) * (x', y', z') = (x + x', y + y', z + z' + xy')$.
- Subgroups of H can be written in the form $\Gamma = M\mathbb{Z}^2 \times m\mathbb{Z}$ where $M \in GL_2(\mathbb{Z})$, $m \in \mathbb{Z}$, and m divides both entries of one row of M.
- Chains of subgroups $\Gamma_n = M_n\mathbb{Z}^2 \times m_n\mathbb{Z}$ can produce normal or subnormal examples.
- E.g., $M_n = \begin{pmatrix} p^n & 0 \\ 0 & p^n \end{pmatrix}$, $m = p$, is a normal chain.

(Several other normal examples in paper by S. Lightwood, A. Şahin and I. Ugarcovici)

- E.g., for distinct primes p, q, $M_n = \begin{pmatrix} p^n & pq^n \\ p^{n+1} & q^{n+1} \end{pmatrix}$, $m = p$. Then we have $N(\Gamma_n) = p\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$ for all n, so the chain is subnormal.
- E.g., for distinct primes p, q, r, $M_n = \begin{pmatrix} rp^n & rpq^n \\ p^{n+1} & rq^{n+1} \end{pmatrix}$, $m = p$.

Then we have $p\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$ contained in $N(\Gamma_n)$, but the normalizer is larger and not the whole group. This chain is subnormal.

- We then associate a Bratteli-type model to these examples.

References

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Joint work with Steve Hurder and Olga Lukina

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