The Algebra Symposium: Extinction II

1. A die is rolled three times. What is the probability of obtaining at least one four?

2. A green and a red dice are rolled once. The number showing in the green die is the x-intercept of a line and the number showing in the red die is the y-intercept of the same line.

   a) What is the probability that the line is parallel to the line $x + y/3 = 1$?

   b) What is the probability that the line is perpendicular to the line $y = x + 1$?

   c) Suppose that the dice are rolled once more. What is the probability that the second line is parallel to the first one?

3. Returning to the extinction problem from our previous symposium. There it was stated that the probability of extinction for a given vertebrate species over a period $T = 5,000,000$ years as equal to 0.1. Denoting by $P\{T = k\}$ the probability that the species becomes extinct during the k-th period, we define the expected extinction time E as:

$$E = P\{T = 1\} + 2P\{T = 2\} + 3P\{T = 3\} + 4P\{T = 4\} + \ldots + nP\{T = n\} + \ldots$$

   Find the expected extinction time remembering that for example

$$P\{T = n\} = (0.9)^{n-1}(0.1)$$

The Merton College Masters (Oxford circa 1330) were able to sum series of the form:

$$1 + 2r + 3r^2 + 4r^3 + \ldots + nr^{n-1} + \ldots$$

The idea was to use the triangular array

$$\begin{aligned} 1 + r + r^2 + r^3 + r^4 + \ldots \\ r + r^2 + r^3 + r^4 + \ldots \\ r^2 + r^3 + r^4 + \ldots \end{aligned}$$

Summing by columns we obtain our original series and summing by rows we obtain the terms of another geometric series.