ASSIGNMENT #36

5.7, #1. $f(x) = x^3 + 3x^2 + 3x - 6$, $f'(x) = 3x^2 + 6x + 3 = 3(x+1)^2$. In particular, $f'(x)$ is never negative so that $f(x)$ is increasing. Thus f has only one root r. Since $f(0) = -6$, $f(1) = 1$, it follows that r is in the interval $(0, 1)$. We start the iterative process $x_{n+1} = x_n - (f(x_n)/f'(x_n))$ with $x_0 = 1$. The truncated values for the next three x_i are

0.9166667
0.9129384
0.9129311

A calculation shows that $f(0.9129311) > 0$ and $f(0.9129300) < 0$. The sign change shows that $0.9129300 < r < 0.9129311$. Thus 0.913 approximates r to 2 decimal places since $|0.913 - r| < 0.913 - 0.9129300 < 1/200$.

5.6, #2. $\sqrt[3]{50}$ is a root of $f(x) = x^3 - 50$. Since $f'(x) = 3x^2$, $f(x)$ is increasing and f has only one root r. Now $f(4) = 14 > 0$, $f(3.5) = -7.125 < 0$, so r is in $(3.5, 4)$. We start the iterative process with $x_0 = 3.5$. The truncated values for the next three x_i are

3.693877
3.684057
3.684031

A calculation shows that $f(3.68406) > 0$ and $f(3.68403) < 0$. The sign change shows that $3.68403 < r < 3.68406$. Thus 3.684 approximates r to 2 decimal places since $|3.684 - r| < 3.68406 - 3.684 < 1/200$.

5.6, #6. $f(x) = \cos x - x$, $f'(x) = -\sin x - 1$. In particular, $f'(x)$ is never positive. So $f(x)$ is decreasing and f has only one root r. Now $f(0) = 1 > 0$, $f(1) = -0.45969\cdots < 0$, so r is in $(0, 1)$. We start the iterative process with $x_0 = 1$. The truncated values for the next three x_i are

0.750363
0.739112
0.739085

A calculation shows that $f(0.73908) > 0$ and $f(0.73911) < 0$. The sign change shows that $0.73908 < r < 0.73911$. Thus 0.739 approximates r to 2 decimal places since $|0.739 - r| < 0.73911 - 0.739 < 1/200$.