ASSIGNMENT #19

3.4, #10. Change in income $= \int_0^{12} r(t)dt = \int_0^{12} 40(1.002)^t dt = \$485.80$

3.4, #14. Notice that each square on the graph represents 10/6, or 5/3 miles. At t=1/3 hours, $v = 0$. The area between the v graph and the t-axis over the interval $0 < t < 1/3$ is $-\int_0^{1/3} vdt$ which is about one square or 5/3 miles. At $t = 1/3$ she is about $5 - 5/3$ or about 10/3 miles from the lake. v is positive on the interval $1/3 \leq t \leq 1$, so she is moving away from the lake on that interval. About 8 squares are between the v graph and the $t-$axis. At $t = 1$,

$$\int_0^1 v\ dt = \int_0^{1/3} v\ dt + \int_{1/3}^1 v\ dt \approx -\frac{5}{3} + 8 \cdot \frac{10}{6} = \frac{35}{3},$$

and the cyclist is about $5 + \frac{35}{3} = \frac{50}{3} = 16\frac{2}{3}$ miles from the lake. Since, starting from the moment $t = \frac{1}{3}$, she moves away from the lake, the cyclist will be farthest away from the lake at $t = 1$. The maximal distance is $16\frac{2}{3}$ miles.

3.5, #2.

$$\lim_{t\to\infty} t^3 e^{-t} = 0$$

A viewing window with $0 \leq x \leq 20$ (or larger x) and $0 \leq y \leq 2$ suggests these limits. (In fact, $t^n e^{-1} \to 0$ as $t \to \infty$ for any $n > 0$. This expresses in another way the fact that exponential functions dominate power functions.)

3.5, #3.

$$\lim_{x\to\infty} \frac{\sin x}{x} = 0$$

A viewing window with $0 \le x \le 20$ (or larger x) and $-2 \le y \le 2$ suggests these limits. It is easy to give a direct argument why the limit is 0. The numerator $\sin x$ is always between -1 and 1 while the denominator x gets arbitrarily large as $x \to \infty$.

3.5, #5. If the exponent were a fixed number, like 3, we could bring the limit inside the expression that is raised to the exponent. However, the exponent n is not fixed, but rather is going to infinity at the same time that the $1+\frac{1}{n}$ is going to one. So we cannot bring the limit inside the exponent.

Now, if we were to ask the calculator to evaluate $(1+\frac{1}{n})^n$ for $n =$ say 10^{20}, it will try to evaluate $1+\frac{1}{10^{20}}$ and it will have no choice but to round off the answer to exactly 1. Then it will take $1^{10^{20}}$, which is 1.