M417

Fall 1996

hw12.tex due Nov 18, 1996

Let D be a simply connected domain. Let z_0 be a fixed point in D. Suppose that $f(z)$ is analytic in D, that $f(z) \neq 0$ in D, and that $f(z_0) = 1$.

1. Show that there is a function $H(z)$ which is analytic in D, $H(z_0) = 0$, and

$$\frac{dH}{dz} = \frac{f'}{f}, z \in D.$$

2. Show that

$$\frac{d\left(f \exp\left(-H\right)\right)}{dz} = 0, z \in D.$$

3. Show that

$$f(z) = \exp\left(H(z)\right), z \in D.$$

4. Show that "f has a square root defined in D", i. e., there is a function $g(z)$, analytic in D, $g(z_0) = 1$, such that

$$\left(g(z)\right)^2 = f(z), z \in D.$$

N.B.: The function $H(z)$ is called a logarithm of the nonzero function f. How does the assumption that D is *simply connected* play an essential role?