

M417

Fall 1996

hw9.tex due Oct. 30, 1996

Let $C = \{|z| = 1\}$, traversed counterclockwise. This exercise treats integrals of the form

$$I(z) = \frac{1}{2\pi i} \oint_C \frac{h(\zeta)}{\zeta - z} d\zeta, \zeta \notin C,$$

for various choices of $h(\zeta)$.

For each choice of $h(\zeta)$, find

- a) $I(z)$, z inside C
- b) $I(z)$, z outside C
- c) $\lim_{r \uparrow 1^-} I(rz_0)$, $z_0 = e^{i\theta_0} \in C$

Choices of $h(\zeta)$:

- 0. $h(\zeta) = h_0(\zeta) = 1 = (\zeta^0)$
- n. $h(\zeta) = h_n(\zeta) = (\zeta^n)$, $n = 1, 2, \dots$
- 1. $h(\zeta) = h_{-1}(\zeta) = \frac{1}{\zeta} = (\zeta^{-1})$
- n. $h(\zeta) = h_{-n}(\zeta) = \zeta^{-n} = \left(\frac{1}{\zeta^n}\right)$, $n = 1, 2, \dots$