Math 165 Linear Price – Demand Model

See Example 1.4.5 on p. 47.

The phrase

for each \$1 increase in the price, 400 fewer cassettes are sold

can be expressed mathematically as

$$\frac{\text{change in demand}}{\text{change in price}} = \frac{-400}{1.00},$$

a constant (for each).

Another statement:

the rate of change of demand with respect to price is $\frac{-400}{1.00}$, a constant

Using the point–slope form for the demand, q, in terms of the price, p,

$$q = q_0 + \overbrace{(-400/1.00)}^{\text{fixed ratio}} (p - p_0),$$

with $q_0 = 4000$, and $p_0 = 5.00$, so that

$$q = 4000 + \frac{-400}{1.00} \left(p - 5.00 \right).$$

We could also restate

for each \$1 increase in the price, 400 fewer cassettes are sold

as

$$\frac{\text{change in price}}{\text{change in demand}} = \frac{1.00}{-400},$$

a constant.

Another statement:

the rate of change of price with respect to demand is $\frac{1.00}{-400}$,

or

Using the point–slope form for the price, p, in terms of the demand, q,

$$p = p_0 + \frac{1.00}{-400} \left(q - q_0 \right),$$

with $q_0 = 4000$, and $p_0 = 5.00$, so that

$$p = 5.00 + \frac{1.00}{-400} \left(q - 4000 \right).$$

Examples

1. for each 0.50 increase in the price, 120 fewer cassettes are sold

$$q = q_0 + \frac{-120}{0.50} (p - p_0),$$

$$p = p_0 + \frac{0.50}{-120} (q - q_0).$$

We could interpret $\frac{p - p_0}{0.50}$ as the *number* of 0.50 price increases.

2. for each 0.25 increase in the price, 140 fewer cassettes are sold

$$q = q_0 + \frac{-140}{0.25} (p - p_0),$$

$$p = p_0 + \frac{0.25}{-140} (q - q_0).$$

3. for each 0.08 increase in the price, 56 fewer cassettes are sold

$$q = q_0 + \frac{-56}{0.08} (p - p_0),$$

$$p = p_0 + \frac{0.08}{-56} (q - q_0).$$

4. for each 0.10 increase in the fare, there are 180 fewer riders

$$q = q_0 + \frac{-180}{0.10} (p - p_0),$$

$$p = p_0 + \frac{0.10}{-180} (q - q_0).$$

5. for each 0.08 increase in the fare, there are 300 fewer riders

$$q = q_0 + \frac{-300}{0.08} (p - p_0),$$

$$p = p_0 + \frac{0.08}{-300} (q - q_0).$$