Math 165: Special Assignment Two - REVENUE COST PROFIT
Due March 10, 2005 at your discussion section

The Rules. Special Assignment Two is a GROUP PROJECT. All papers must be worked on and written up by groups of at least two and no more than four people. TYPE your writeup. The last paragraph should summarize the roles and activity of each group member. Groups may assign tasks, but each member is responsible for explaining any part of the assignment.

WARMUP

W1. Differentiate with respect to $x$: $p(x) = \frac{1}{x^2 + 1}$.
You may TYPE the expression as you might enter it on your calculator: $p(x) = 1/(x^2 + 1)$.

W2. Differentiate with respect to $x$: $r(x) = \frac{x}{x^2 + 1}$. Simplify your answer before proceeding.

W3. Find the critical numbers and critical points of the graph of $r(x)$.

W4. Use your calculator to draw the graph of $r(x)$ and $r'(x)$. A window of $0 \leq x \leq 10$, $-1 \leq y \leq 1$, is suggested. Your graph should look like this:

\begin{center}
\begin{tikzpicture}
\begin{axis}[
    title={},
    xlabel={$x$},
    ylabel={$y$},
    xmin=-5, xmax=5,
    ymin=-5, ymax=5,
    xtick={},
    ytick={},
    axis lines=middle,
    enlarge y limits=true,
    samples=100,
    no markers,
    domain=-5:5,
    legend pos=south east
]
\addplot[color=red, thick] {x/(x^2+1)} node[above] {$r(x)$};
\addplot[color=green, thick] {1/(x^2+1)} node[below] {$r'(x)$};
\end{axis}
\end{tikzpicture}
\end{center}

Notice that the graph of $Y1 = r$ has its MAXimum at the ROOT of $Y2 = r'(x) = 0$.

W5. Now practice finding the root of the equation $Y2 = 0$ by using your calculator. You should use several methods. Try TRACE AND ZOOM on the graph of $Y2$. If available, use the ROOT or SOLVE or INTERSECT facility on your calculator. See your calculator manual. There are also several short manuals for TI calculators on the Blackboard Course Site under COURSE DOCUMENTS/Graphing Calculator Workshop.
W6. Once you have found a good method to solve the equation \( r'(x) = 0 \), go back and find the critical numbers and MAXimum value for the function

\[
Y_1 = \frac{x}{x^2 + 1} - 0.1x \\
= r(x) - 0.1x
\]

on the interval \( 0 \leq x \leq 10 \). Use \( Y_2 = r'(x) - 0.1 \). The solution should be about \( x = .84 \). Your graph should look like this:

![Graph of r(x) and r'(x) with critical points]

Main Course: REVENUE COST PROFIT

This is based on Chapter 2 Section 2.3 Problem 44 p. 132 DEMAND AND REVENUE.

The manager of a company that produce graphing calculators determines that when \( x \) thousand calculators are produced, they will be sold when the price is

\[
p(x) = \frac{1,000}{0.3 * x^2 + 8}
\]
dollars per calculator.

(a) At what rate is price \( p(x) \) changing with respect to the level of production \( x \) when 3,000 \( (x = 3) \) calculators are produced?

(b) The revenue in thousands of dollars derived from the sale of \( x \) thousand calculators is

\[
Revenue(x) = x * p(x).
\]

We introduce a cost function. Assume that there is a fixed cost of 25000 dollars each month and that each calculator requires 35 dollars in parts, labor, etc.

The cost function in thousands of dollars per month is

\[
Cost(x) = 25.0 + 35.0 * x.
\]
The profit in thousands of dollars per month in terms of a production of \( x \) thousand calculators per month is

\[
\text{Profit}(x) = \text{Revenue}(x) - \text{Cost}(x).
\]

(c) Graph the functions \( \text{Revenue}(x) \), \( \text{Cost}(x) \), and \( \text{Profit}(x) \). A window of \( 0 \leq x \leq 10, -100 \leq y \leq 400 \) is suggested. Your graph should look like this:

(d) (BREAK EVEN POINT) What is the minimum number of calculators produced each month which assure a profit?

Find the answer \( x \) to the nearest 0.1 (the nearest 100 calculators). Use the calculator methods you know to solve the equation \( \text{Profit}(x) = 0 \) or \( \text{Cost}(x) = \text{Revenue}(x) \).

I suggest using TRACE and ZOOM the your calculator graph(s) to estimate the solution. If SOLVE is available on your calculator, find its syntax and investigate the solution using SOLVE. Be careful! There is a negative “solution” which is “not practical.”

What is the price of a calculator (nearest cent) at the break even level of production?
(c) (MAXIMUM PROFIT) Estimate the monthly level of production x (nearest 0.1 thousand or 100 calculators) which yields the maximum profit.

The way to maximize the Profit is to find the critical point the marginal profit:

\[
\frac{d\text{Profit}}{dx}
\]

Find a formula for the marginal profit function and solve the equation

\[
\frac{d\text{Profit}}{dx} = 0.
\]

Graph the marginal profit function. A window of \(0 \leq x \leq 10, -100 \leq y \leq 100\). is suggested. Your graph should look like this:

I suggest using the methods you know to solve the equation \(\text{Profit}'(x) = 0\).

What is the price of a calculator at the at maximum profit (nearest cent)?

What is the maximum profit (nearest thousand dollars)?

(f) Is a monthly production of 10000 calculators profitable? Estimate the maximum number of calculators produced which would assure a profit.

**Remember the Rules**

The last paragraph of your typed writeup should summarize the roles and activity of each group member.
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0. For members of your group, list names, TAs, and discussion time.

1.

2.

3.

4.

WARMUP

W1. Write \( p(x) \). By the ??? rule \[ \text{[formula]} \] \( \frac{dp}{dx} = \ldots \).

You may TYPE the expression as you might enter it on your calculator: \( p(x) = \frac{1}{(x^2 + 1)} \).

W2. Write \( p(x) \). By the ??? rule \[ \text{[formula]} \] \( \frac{dr}{dx} = \ldots \). The simplified answer is \( \ldots \).

W3. The critical numbers for \( r \) are [definition]. Solving the equation ??? as follows, the critical numbers are \( \ldots \).

The critical points on the graph are ( , ) and ...

W4. Using a ??? calculator, we graphed the two functions .... with \( Y_1 = \ldots \), \( Y_2 = \ldots \), to obtain the attached graph...

Notice that the graph of \( Y_1 = r \) has its MAXimum at the ROOT of \( Y_2 = r'(x) = 0 \), as is shown by the indicated vertical line.

W5. We found several different ways to solve the equation \( Y_2 = 0 \). ...

W6. We solved the equation ... by using .... Describe in detail the method. We checked by using another method (details) or plugging our answer back into the equation (details).

Main Course: REVENUE COST PROFIT

This is based on Chapter 2 Section 2.3 Problem 44 p. 132 DEMAND AND REVENUE. The manager of a company that produce graphing calculators determines that when \( x \) thousand calculators are produced, they will be sold when the price is

\[
p(x) = \frac{1,000}{(0.3 \times x^2 + 8)}
\]
dollars per calculator.
(a) At what rate is price $p(x)$ changing with respect to the level of production $x$ when 3,000 ($x = 3$) calculators are produced?

We calculated ....

(b) The revenue in thousands of dollars derived from the sale of $x$ thousand calculators is

$$\text{Revenue}(x) = x \cdot p(x).$$

We introduce a cost function. Assume that there is a fixed cost of 25000 dollars each month and that each calculator requires 35 dollars in parts, labor, etc.

The cost function in thousands of dollars per month is

$$\text{Cost}(x) = 25.0 + 35.0 \cdot x.$$

The profit in thousands of dollars per month in terms of a production of $x$ thousand calculators per month is

$$\text{Profit}(x) = \text{Revenue}(x) - \text{Cost}(x).$$

(c) Graph the functions Revenue$(x)$, Cost$(x)$, and Profit$(x)$. A window of $0 \leq x \leq 10$, $-100 \leq y \leq 400$ is suggested.

The formulas (with units, for Revenue, Cost, and Profit are

Profit ($\$1000 \text{ per month}$) = ...

(d) (BREAK EVEN POINT) What is the minimum number of calculators produced each month which assure a profit?

The producer “breaks even” when ... calculator are sold monthly at a price of .... This is a result of the calculations ....

We solved the equations using ..... We checked the answer by ...

(e) (MAXIMUM PROFIT) Estimate the monthly level of production $x$ (nearest 0.1 thousand or 100 calculators) which yields the maximum profit.

We graphed the monthly Profit function using $Y1 = ...$

The marginal profit, $d\text{Profit}/dx$ is .... We investigated the graph[s] of ...

It seems that if we sell .... calculators at ... dollars apiece, we obtain a maximum monthly profit of ...

(f) Is a monthly production of 10000 calculators profitable?

Well, it seems that our profit would be ...., so ...

**Remember the Rules**

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