

## Math 165 Spring 2009 SA3: Pricing Annuities

Due Friday April 24, 2009, in Lecture

### The Rules

- This assignment should be **typed**. For suggestions on typing, see <http://www2.math.uic.edu/~jlewis/math165/165type.pdf>
- Special Assignment Three is a **GROUP PROJECT**. All papers must be worked on and written up by groups of at least two and no more than four people. **TYPE** the group writeup. **Please use complete sentences to explain your work and answers.** For graphs, you may attach neat free hand sketches with enough labels for an outsider to understand the graph. Groups may assign tasks, but each member is responsible for understanding all parts of the assignment. The last paragraph should summarize the roles and activity of each group member.
- Please note: Using the results of another group without acknowledgment is considered academic dishonesty and will be severely punished.
- Turn in the **Antipasto, Main Course, and Dessert.**

### Present Value and Future Value of a Continuous Income/Investment Stream

See Hoffmann, pp. 422–425.

For SA3 we shall assume an “investment” or “account” earns at an annual rate of  $r = 3.7\%$  CC (compounded continuously).

### Antipasto – Turn In!

A1. Money is deposited continuously in an account at a rate of \$1000 per year, what is the value of the account after  $T = 15$  years?

**Typing suggestion:** Borrowing from Maple or your calculator, you may type  $\int_a^b f(t) dt$  as

`int(f(t), t = a .. b).`

or

`fnInt(f(t),t,a,b).`

A2. Money is deposited continuously in an account at a rate of \$1000 per year for a period of  $T$  years. At the end of  $T$  years the value of the account is \$10000. What is the value of  $T$ ?

Your solution should include a sentence of the form: **Solve the equation ... = ... for  $T$  ...**

A3. I have just enough money in an account so that money may be withdrawn continuously at a rate of \$1000 per year for  $T = 15$  years. What is the present value of the account?

A4. My account starts with \$10000, and money is withdrawn at a rate of \$1000 per year for  $T$  years. What is the value of  $T$ ? (When does the account have 0 balance?)

## Main Course

**Please use complete sentences to explain your work and answers.**

M1. Beginning at age 65, I wish to withdraw \$12000 per year for 20 years. What should be the value of the account when I am 65? Call this number  $A_{65}$  – the assets needed at age 65 to “buy” the annuity.

M2. If I start contributing (continuously of course) to the account at age 55, at what annual rate must I contribute to have the value  $A_{65}$  at age 65?

M3. If I start contributing (continuously of course) to the account at age 45, at what annual rate must I contribute to have the value  $A_{65}$  at age 65?

M4. I can contribute only at the rate of \$6000 per year. At what age do I need to start contributing to have the value  $A_{65}$  at age 65?

M5. I can contribute only at the rate of \$3000 per year. At what age do I need to start contributing to have the value  $A_{65}$  at age 65?

### **Dessert: Annuities with COLA (Cost of Living Adjustment)**

Assume that inflation is occurring at 2% per year CC.

COLA1. Beginning at age 65, I wish to withdraw from an account for 20 years. I will begin the withdrawals at the rate of \$12000 per year. The payments will have a COLA (cost of living adjustment) so that at age  $65 + t$  years I will be withdrawing at a rate of  $\$12000 * e^{.02t}$  per year. What should be the value of the account when I am 65? Call this number  $ACOLA_{65}$  – the assets needed at age 65 to “buy” the annuity with the COLA.

COLA2. At age 45, I start contributing (continuously of course) to the account. My initial rate is \$R45 per year, which I continually adjust for inflation so that at age  $45 + t$  years I will be contributing at rate of  $\$R45 * e^{.02t}$  per year. At what initial annual rate  $\$R45$  must I contribute to have the value  $ACOLA_{65}$  at age 65?

### **Remember the Rules**

The last paragraph of your typed writeup should summarize the roles and activity of each group member.