Show Me the Solution

A manufacturer determines that when $x$ hundred units of a particular commodity are produced, they can all be sold for a unit price given by the demand function $p = 80 - x$ dollars. What is the maximum revenue (in dollars)?

The revenue derived from producing $x$ hundred units and selling them all at $80 - x$ dollars is $R(x) = x(80 - x)$ hundred dollars. Note that $R(x) \geq 0$ only for $0 \leq x \leq 80$. The graph of the revenue function

$$R(x) = x(80 - x) = -x^2 + 80x$$

is a parabola that opens downward (since $A = -1 < 0$) and has its high point (vertex) at

$$x = \frac{-B}{2A} = \frac{-80}{2(-1)} = 40$$

Thus, the revenue is maximized when $x = 40$ hundred units are produced, and the corresponding maximum revenue is

$$R(40) = 40(80 - 40) = 1600$$

hundred dollars. The manufacturer should produce 4000 units and at that level of production, should expect a maximum revenue of $160000.$