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Test One Shistla in LC C1.
Colkers Regular Lecture Ro
Approx by Increments (Tangent Line Linear Appr.
near $x = x_0$

$$f(x_0+h) \approx f(x_0) + \underbrace{f'(x_0)\cdot(x-x_0)}_{\text{approx change} \approx f(x_0+h)-f(x_0)}$$

$$f(x_0+\Delta x) \approx f(x_0) + f'(x_0)\cdot\Delta x$$

Differentials (p. 158)
$y = f(x)$ for definiteness

The differential of y, $dy \equiv \frac{dy}{dx}\cdot dx$ OR $f'(x)dx$

Meaning. If dx is replaced by small Δx

$$\Delta f \approx f'(x)\,\Delta x \;\; \lim_{\Delta x \to 0} \frac{\Delta f - f'(x)\Delta x}{\Delta x} = 0$$

Differential + Chain Rule

$$\frac{d}{dx} f(u) = \frac{df}{du}\cdot\frac{du}{dx}$$

$$d[f(u(x))] = \underbrace{f'(u(x))}_{\text{deriv with}}\, du(x) = f'(u(x))\cdot \frac{du}{dx}\, dx$$

Repeated from

$$d(u^p) = p\,u^{p-1}\,du = \underbrace{p\,u^{p-1}\frac{du}{dt}}_{\frac{du^p}{dt}}\,dt$$

If $u = u(t)$ $du = \frac{du}{dt}\,dt$

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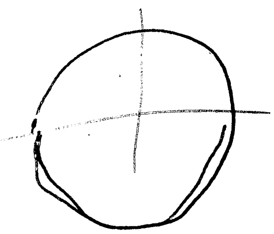
Function given implicitly by RELATION between x, y

GEOMETRY Circle $x^2+y^2=25$



Not a function but there are two functions to be considered

[Solve for y in terms of x, ... $y=\pm\sqrt{1-x^2}$

$y^3+4xy \cancel{+10=0} = \underbrace{16}$

$x=1, y=2$

$y(y^2+4x) = \_\_$

$2^3+4\cdot1\cdot2=16.$

Explicit. Solving a cubic

(p.163) $x^2y+2y^2=3x+2y$

but suppose there is a nice (diff fn) s.t. $y=f(x)$ satisfies the equation EVEN if don't know it explicitly!)

We use our rules [~~sum~~ $\pm$, $\cdot$, $\div$, CHAIN]

$\boxed{\begin{array}{c}2.6.1\\ p.\ 164\end{array}}$ $x^2y+y^2=x^3$

$$2x\cdot y + x^2\underbrace{\frac{dy}{dx}}_{\text{diff } y \text{ [no form]}} + 2y\cdot\frac{dy}{dx} = 3x^2$$

SOLVE for $\frac{dy}{dx}$ [POSSIBLE

$$(x^2+2y)\frac{dy}{dx} = \cancel{x^2-2xy}\ x^2-2xy$$

$$\frac{dy}{dx} = \frac{3x^2-2xy}{x^2+2y}$$ (of course $x^2+2y\neq0$)!

Can differentials work?

$$x^2 y + y^2 = x^3$$

(BTW can be explicit notion)

$$\underbrace{1}_{A} y^2 + \underbrace{x^2}_{B} y + \underbrace{(-x^3)}_{C}$$

$$y = \frac{-1 \pm \sqrt{\cancel{x}^2 - 4(-x^3)}}{2}$$

Two functions

$$x^2 + 4x^3 \geq 0$$

PROD RULE $\frac{duv}{dx} = \frac{v\,du}{dx} + \frac{u \cdot dv}{dx}$

$$d(x^2 y) + d\,y^2 = dx^3$$

$$\downarrow$$

$$\frac{2x\,dy}{dx} + \frac{y \cdot 2x\,dx}{dx} + \frac{2y\,dy}{dx} = \frac{3x^2\,dx}{dx}$$

Differential is the "thing" that dividing by dx, etc gives the right answer.

(No non mathematician ever lost her job for misusing differentials.)

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2.6.2: Slope of tangent line $\left[\frac{dy}{dx}\right]$! Symbol

$x^2+y^2=25$ at $x=3,4$, $y= \begin{matrix} 4 \\ -4. \end{matrix}$

Ex 2.6.4

Output Q satisfies

$$Q = 2x^3 + x^2 y + y^3$$

$x =$ hrs skilled

$y -$ hrs unskilled

Now $x=30$, $y=20$

Change in y to offset 1 hr increase in x

$$\boxed{dQ=0}$$

$$8000 = 2x^3 + x^2 y + y^3$$

$$\Delta Q \approx dQ = 6x^2 dx + \underline{2xy\, dx + x^2 dy} + 3y^2 dy$$

Want $dQ=0$ (maintain same output)

$$6x^2+2xy)dx + (x^2+3y^2)dy$$

$$\frac{dy}{dx} = - \text{———} \approx -3.14$$