Elasticity – See Notes

Applications: Revenue, et al 3.5 20090227 1/

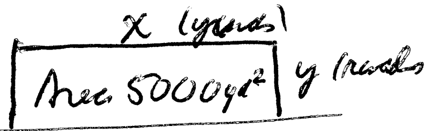
Example 3.5.1. (Pasturage Problem

Park, Rectangular area 5000 (yds)^2

~~Lea~~ bounded on 3 sides

— Minimum length of fencing

— How long and wide



$xy = 5000$

$L(x,y) = 2y + x$

"Minimize L"; L has two variables, need one

$x = \frac{5000}{y}$ $L = L(y) = 2y + \frac{5000}{y}$ $0 \le y \le$ ~~[illegible]~~

* practical domain (drew several)

$\frac{dL}{dy}$

$\frac{dL}{dy} = 2 - \frac{5000}{y^2}$; Solve $=0$ ... $y^2 = \frac{5000}{2} = 2500$

Let $y = 50$ $(\sqrt{2500})$ and $x = \frac{5000}{y} = \frac{5000}{50} = 100$

Why Min

2500 y

Th~~e~~ Many (usually two) variables x, y ~~=5~~

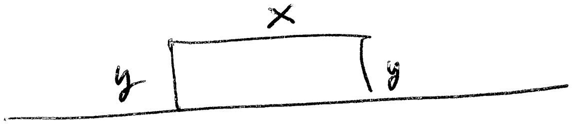
Quantity to be Optimized in terms of x, y

Reduce to one variable

Apply Calculus meth $\frac{dw}{dw} = 0$

What was asked

Same problem



$xy = 5000$

$L = x + 2y.$

$\boxed{y = \frac{5000}{x}}$

$L = L(x) = x + \frac{2\cdot 5000}{x}$ $x > 0$

$\frac{dL}{dx} = 1 - \frac{10000}{x^2}$, ... $x^2 = 10000$, $x = 100$..

$y = 50$

Minimum $L = x + 2y\big|_{x=100} = 100 + 2\cdot 50 = 200$

Retail Sales – Maximize Profit

At price of \$40 $x = 50$ sales per month

"for each \$1 increase in price, 3 less units sold each month

Each unit costs 25 to produce"

Find price at which profit is maximized

$P = p\cdot x$ ← two variables

$\cancel{\frac{dP}{dp} = 1\cdot x + p\cdot \frac{dx}{dp}}$ $\cancel{\frac{dx}{dp} = -3}$

$\cancel{\frac{dP}{dp} = 0 \text{ when } x - 3p = 0}$

Take into account Cost $= 25x$

(*) $P = \cancel{(x-25)}$ $\cancel{x(p-25)}$ Express profit reasonably

$\boxed{\text{Here}}$ $x - \underbrace{50 = -3}_{\frac{dx}{dp}}(p - 40)$

$x = 170 - 3p$

$x = 50 - 3p + 120 = \cancel{90 - 3p}$

$P(p) = \cancel{x\cdot p = 90p - 3p^2}$

$P(p) = p(170 - 3p) = 170p - 3p^2$

$\frac{dP}{dp} = 170 - 6p$, $p = \frac{170}{6} \approx$

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$P = x(p-25)$

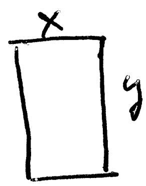
$x - 50 = \underbrace{\frac{dx}{dp}}_{-3}(p-40)$: $x = 50 - 3p + 120$
$= 170 - 3p$

$P = (170-3p)(p-25)$

$\frac{dP}{dp} = -3(p-25) + 170 - 3p$
$= -6p + 75 + 170$; $p = \frac{170+75}{6} \approx$
≈ 40.83

3.5 probs

$\boxed{9}$ Rectangular Playground are of $3600\ ft^2$ surround by fence "How can this be done"



$xy = 3600$

minimize $L(x,y) = 2x + 2y$

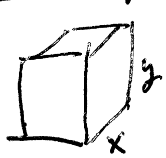
$y = \frac{3600}{x}$

$L = L(x) = 2x + 2 \cdot \frac{3600}{x}$

$\frac{dL}{dx} = 2 - \frac{2 \cdot 3600}{x}$; $= 0$ when... $x = 60$

$y = \frac{3600}{x} = 60$. Let $x = y$ or MAKE A SQUARE!

Problem 15 Open box with a square base
~~bottom at 3/m²~~, sides $3/m^2$
base $4/m^2$



Minimize ~~C(x,y) Find dimens~~ greatest volume with fixed

cost of 48

$C(x,y) = x^2 + 4xy = 48$

Maximize $V = x^2 y$

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$\dots\ y = \frac{48-x^2}{4x}$

$V = x^2\left(\frac{48-x^2}{4x}\right) = \frac{x(48-x^2)}{4} = \frac{48x-x^3}{4}$

$\cancel{\frac{dy}{}}\ \frac{d\cancel{y}V}{dx} = \frac{1}{4}(48-3x^2), \dots$ Let. $x^2 = \frac{48}{3}$, $\boxed{x=4}$

and $y = \frac{48-16}{4\cdot 4} = 2$

$\boxed{N.B}$ $C = x^2 + 4xy = 16 + 4\cdot 4\cdot 2 = 48$