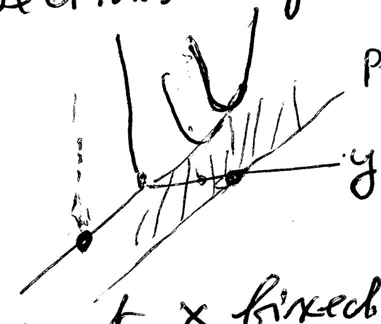
SUMMARY Several Variables 20090420

$z = F(x,y) = x^2 + 2y^2$

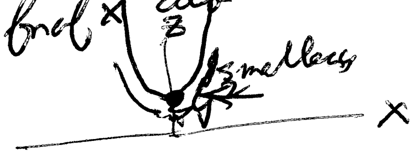
Full Graph

Cross Sections: $y =$ constant (pos fore



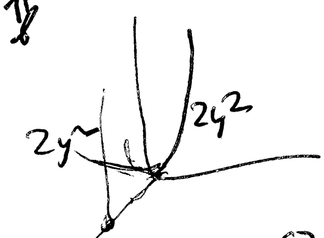

$f(x,y) = x^2 + 2y^2$

Cas fixed $x$ alt



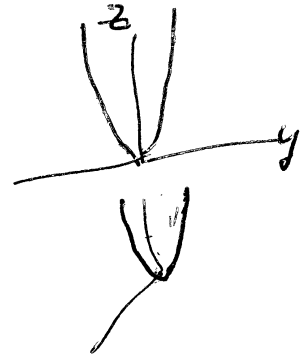

When looking at $x$ fixed

$f(\bar{x},y) = x + y^2$




In particular $x=0$

$y=0$




Graphs —

Partial Derivatives

Quadratic $Ax^2 + Bxy + Cy^2 + Dx + Ey + F$

Fns of several variables (usually 2 or 3)   20090420   1

- Functions of Two variables $(x, y)$   $[(p_1, p_2), (q_1, q_2)]$

Maximize Revenue from selling two products

Graph a fnd

$$q_1 = A - Bp_1 + Cp_2$$

(~~lowering p~~ raising $p_1$ decreases demand for $q_1$

~~Raising price for q~~ Raising price $p_2$ raises demand for $q_1$

$$q_2 = D \cancel{+ Ep_2 - F}$$
$$= D + Ep_1 - Fp_2$$

$$R = p_1 q_1 + p_2 q_2$$
$$= -Bp_1^2 + Cp_1p_2 + Ep_1p_2 - Fp_1^2$$

$R_{11} = -2B \quad R_{12} = R_{21} = (C+E) \quad R_{22} = -2F$

Discriminant $\cancel{R_{12}^2 - R_{11}R_{22} - (C+E)^2 + 4BF} > 0$

$D = R_{11}R_{22} - R_{12}^2$

[redacted]

Temperature at T(x,y)        (T, x, y, c)  20090420    2/

"150" therms

Partial Derivatives of $x, y$    $F(xy) = (x^2 - 4xy + y^2)$

"p.d. wrt. x" FREEZE y (y is perma

$$\frac{F(x+dx, y) - F(x,y)}{dx}$$

(Actually means - let $(\Delta x) > 0$, small

Take lim as $\Delta x \to 0$

"partial derivative of F wrt y

Slices of a fn where $y =$ constant

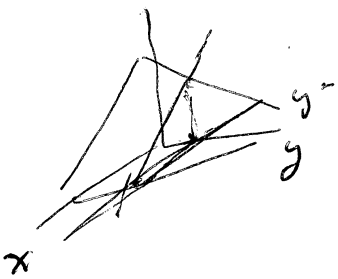



Fig 7.4   z plane

Example Utility Function

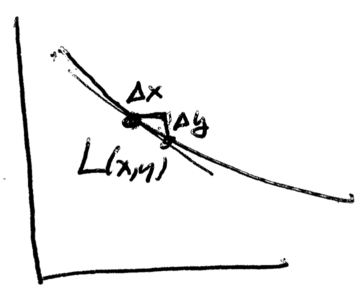
$$L(x,y) = x^{3/2} y$$

$$L(16,20) = 16^{3/2} \cdot 20$$ (= 1280 by hand or calculator

Level Curve through (16,20)

graphed

Implicit Differentiation




$$\partial L(x,y) = x^{3/2} y$$

$$\frac{\partial L}{\partial x} = \frac{3}{2} x^{1/2} y \quad \frac{\partial L}{\partial y} = x^{3/2}$$

treat y as const

How does want do $\frac{dy}{dx}$:

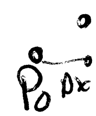
$$\frac{d}{dx}\left(x^{3/2} y\right) = \left(\frac{3}{2} x^{1/2} y\right) + x^{3/2} \frac{dy}{dx} = \frac{\partial L}{\partial x} + \frac{\partial L}{\partial y}\frac{dy}{dx}$$

$\Delta F$

changing $x$ by $\Delta x$ ] both small (multiple)
$y$ by $\Delta y$

$\Delta F$ $\quad F(x+\Delta x, y+\Delta y) \approx \frac{\partial F}{\partial x}\Delta x + \frac{\partial F}{\partial y}\Delta y$

$$= F(x+\Delta x, y) - F(x,y) + F(x+\Delta x, y+\Delta y) - F(x+\Delta x, y)$$



$$\approx \left.\frac{\partial F}{\partial x}\right|_{(x,y)} \Delta x + \left.\frac{\partial F}{\partial y}\right|_{(x,y)} \Delta y$$

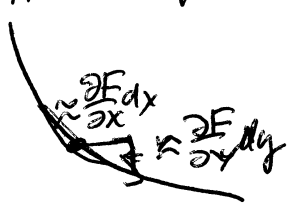
The differential of $F(x,y)$ is

$$dF \equiv \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy$$

If $x, y$ depend on a variable $s$

$$\frac{dF}{ds} = \frac{\partial F}{\partial x}\frac{dx}{ds} + \frac{\partial F}{\partial y}\frac{dy}{ds}$$

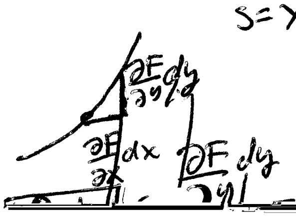
In particular if $x(t), y(t)$ passes along a Level curve




$$\frac{dF}{ds} = 0 \qquad \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y}\frac{dy}{dx} = 0$$

$\vec{s} = x$

$$\boxed{\frac{dy}{dx} = -\frac{\partial F/\partial x}{\partial F/\partial y}}$$




$F = \text{const.}$ $\quad$ Change in $F = 0$

[redacted]

On an ISOQUANT. How much does y change when x changes by a small amount



Example 7.2.10

$$Q = Q(K,L) = 60K^{1/2}L^{1/3}$$

K capital ($1000)

L: worker hours

When $K = 100$, $L = 1000$ ← add 1 ← add 2 $(=10^3)$

$L^{1/3} = 10$

$$\Delta Q \approx \frac{\partial Q}{\partial K}\Delta K + \frac{\partial Q}{\partial L}\Delta L$$

When $Q = 600$ $L = 1000$

$$\frac{\partial Q}{\partial K} = 60\left(\frac{1}{2}K^{-1/2}\right)L^{1/3}$$

$$\frac{\partial Q}{\partial L} = 60\left(K^{1/2} \cdot \frac{1}{3}L^{-2/3}\right.$$

$$\frac{\partial Q}{\partial K} = 60 \cdot \left(\frac{1}{2} \cdot \frac{1}{30}\right) = 1$$

$$\frac{\partial Q}{\partial L} = 60 \cdot 30 \cdot \frac{1}{100} = 18$$

What is $\frac{\partial L}{\partial K}$ $-\frac{1}{18}$