So: Along a "level curve of F where y is a fn of x

$$\frac{\partial F}{\partial x} + \frac{\partial F}{\partial y}\frac{dy}{dx} = 0.$$

~~Ob~~ Optimizing a function of Two variables
(MAX)

(x_0, y_0) is a relative / local maximum for $f(x,y)$

if for (x,y) near (x_0,y_0) $F(x_0,y_0) \geq F(x,y)$

~~F~~ At a rel max [where differen m

$$\frac{\partial F}{\partial x} = 0 \; ; \; \frac{\partial F}{\partial y} = 0 \quad \text{CRITICAL POINT}$$

[2 ~~of~~ equations in 2 unknowns

[Subst Commodity: ~~Special~~ Lead Gas and Premium Gas

Complementary: Demand for both inc / dec. together

$\boxed{27}$ q_1 $D_1 = 500 - 6p_1 + 5p_2$ } substitute
q_2 $D_2 = 200 + 2p_1 - 5p_2$ } Commodities

Revenue $p_1 q_1 + p_2 q_2$

$$= p_1(500 - 6p_1 + 5p_2) + p_2(200 + 2p_1 - 5p_2)$$

Possible to maximize

$$\frac{\partial R}{\partial p_1} = -12p_1 + 5p_2 + 500$$

$$\frac{\partial R}{\partial p_2} =$$

Notice Quadratic in p_1, p_2 2009 04 ~~24~~ 8/

Quadratics:

$$z = Ax^2 + 2Bxy + Cy^2 + Dx + Ey + F$$

$$\left.\begin{aligned} \frac{\partial z}{\partial x} &= 2Ax + 2By + D \\ \frac{\partial z}{\partial y} &= 2Bx + 2Cy + E \end{aligned}\right\} \text{Solve two linear equations}$$

$$z_{xx} = 2A \quad z_{xy} = 2B = z_{yx} \qquad z_{yy} = 2C$$

At Local max  $z_{xx} < 0$ also $z_{yy} < 0$

$\to x$

Every direction?

discriminate $D(x,y) \equiv (z_{xx} z_{yy}) - (z_{xy})^2$

Cases $D(x,y) > 0$ Loc MAX [MIN] according to z_{xx} (z_{yy})

$D(x,y) < 0$ MAX in some direction MIN in others

$D(x,y) = 0$ Not sure

Notice for quadratics
Discriminant $= (2A)(2C) - (2B)^2$
related to "$B^2 - 4AC$" in quadratic formula

~~7~~.2.23 $f(x,y) = e^{x^2y}$

$f_x = 2xy\, e^{x^2y}$ $\quad f_y = \cancel{2xe}\, x^2 e^{x^2y}$

$(f_x)_y = 2x\, e^{x^2y} + 2xy \cdot x^2 e^{x^2y}$

$(f_y)_x = 2x\, e^{x^2y} + x^2(2xy) e^{x^2y}$ SAME

$f_{yy} = x^2 x^2 e^{x^2y}$.

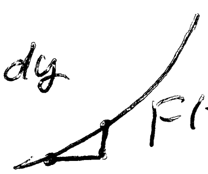
Partials and implicit dv

$$dF = \frac{\partial F}{\partial x}\Big|_{(x,y)} dx + \frac{\partial F}{\partial y}\Big|_{(x,y)} dy$$

If ~~y=~~ $F(x,y) = $ Const and $y = y(x)$

Along the level curve

$$dF = \frac{\partial F}{\partial x} dx + \frac{\partial F}{\partial y} dy$$



$F(x,y)$

≈ 0

or divide by Δx (dx)

$$\boxed{\frac{\partial F}{\partial x} + \frac{\partial F}{\partial y}\frac{dy}{dx} = 0}$$

Example $\quad x^3 y + 2xy^2 = $ Const

Implicit, diff

$$3x^2y + x^3\frac{dy}{dx} + 2y^2 + 2x2y\frac{dy}{dx} = 0$$

$$(3x^2 + 2y^2) + (x^3 + 4xy)\frac{dy}{dx} = 0$$

N.B. $\frac{\partial F}{\partial x} = (3x^2y + 2y^2)$

$\frac{\partial F}{\partial y} = (x^3 + 4xy)$

(use $Ax^2 + Bxy + Cy^2$ etc