165opt.mw
Maple 10 Worksheet Math 165 165opt.pdf- Calculus for Business.

```
> restart:with(plots):with(student):with(plots):with(plottools):
```

Problem 9e.3.4.39

```
> eqn_39:= proc(p,q);
     q^2 + 3*p*q =22:
  end proc:
  `eqn_39(p,q) `:= eqn_39(p,q);
```

$$eqn\_39(p,q) := q^2 + 3\,q\,p = 22 \quad (1)$$

```
> dq_dp:=proc(p,q);
      implicitdiff(eqn_39(p,q),q,p);
  end proc:
  `dq_dp `:=dq_dp(p,q);
  E_39(p,q):=(p/q)*dq_dp(p,q);
  Ans_39:=eval(E_39(p,q),[p=3,q=solve(eqn_39(3,q),q)[1]]);
```

$$dq\_dp := -\frac{3\,q}{2\,q+3\,p}$$

$$E\_39(p,q) := -\frac{3\,p}{2\,q+3\,p}$$

$$Ans\_39 := -\frac{9}{13} \quad (2)$$

Problem9e.3.4.22

{\bf Problem 3.4.22} Price p and demand q
are related by the equation $p = 81 - 3q$. The total cost of
producing q units is $C(q) = (q + 1)/(q + 3)$
\itemitem{(a)} revenue function $R(q) =?$
\itemitem{(b)} profit function $P(q) =?$
\itemitem{(c)} marginal revenue function $R'(q) =?$
\itemitem{(d)} marginal cost $C'(q) =?$
\itemitem{(e)} average cost $A(q) = (C(q)/q) =?$
\itemitem{(f)} derivative of average cost $A'(q) =?$
\itemitem{(g)}Find the q for which $A(q)$ is minimized. For this q, compare
the marginal cost $C'(q)$ and $A(q)$.

```
> p_22:=proc(q);
     81 - 3*q;
  end proc:
  `p_22(q) `:=p_22(q);
  C_22:= proc(q);
    (q + 1)/(q + 3);
  end proc:
```

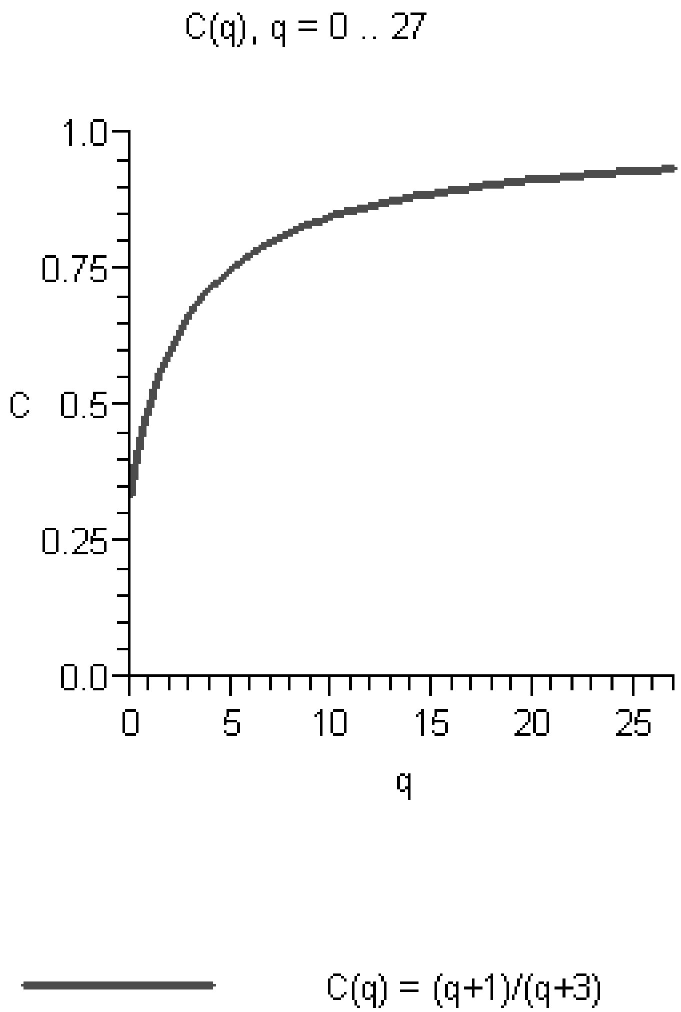
```
`C_22(q) `:=C_22(q);
```

$$p\_22(q) := 81 - 3\,q$$

$$C\_22(q) := \frac{q+1}{q+3} \tag{3}$$

To have the price p = 81 - 3q, positive, we must have 0 \leq p < 27.

```
> plot22:=plot(C_22(q),q= 0 .. 27, C= 0 ..1,thickness = 2,legend=`C
  (q) = (q+1)/(q+3)`,
  title=`C(q), q = 0 .. 27`):
  display(plot22);
```



Notice that geometrically, the average cost would be minimized at the right endpoint: q = 27. Since an endpoint is a SPECIAL
point, we do not expect to satisfy the "criterion to minimize average cost."

\itemitem{(a)} revenue function $R(q) =?$

```
> R_22:=proc(q);
    q*p_22(q);
  end proc:
```

```
`R_22(q) `:=R_22(q);
```

$$R\_22(q) := q\ (81-3\ q) \tag{4}$$

\itemitem{(b)} profit function $P(q) =?$

```
> P_22:=proc(q);
     R_22(q) - C_22(q);
  end proc:
  `P_22(q) `:= P_22(q);`normalized `:=normal(P_22(q));
```

$$P\_22(q) := q\ (81-3\ q) - \frac{q+1}{q+3}$$

$$normalized := -\frac{-72\ q^2 - 242\ q + 3\ q^3 + 1}{q+3} \tag{5}$$

\itemitem{(c)} marginal revenue function $R'(q) =?$

```
> MR_22:=proc(q);
    diff(R_22(q),q);
  end proc:
  `MR_22(q) `:=MR_22(q);
```

$$MR\_22(q) := 81-6\ q \tag{6}$$

\itemitem{(d)} marginal cost $C'(q) =?$

```
> MC_22:=proc(q);
    diff(C_22(q),q);
  end proc:
  `MC_22(q) `:=MC_22(q);`normalized `:=normal(MC_22(q));
```

$$MC\_22(q) := \frac{1}{q+3} - \frac{q+1}{(q+3)^2}$$

$$normalized := \frac{2}{(q+3)^2} \tag{7}$$

\itemitem{(e)} average cost $A(q) = (C(q)/q) =?$

```
> A_22:= proc(q);
    C_22(q)/q;
  end proc:
  `A_22(q) `:=A_22(q);
```

$$A\_22(q) := \frac{q+1}{(q+3)\ q} \tag{8}$$

\itemitem{(f)} derivative of average cost $A'(q) =?$
\itemitem{(g)}Find the q for which $A(q)$ is minimized. For this q, compare the marginal cost $C'(q)$ and $A(q)$.

```
> MA_22:=proc(q);
    diff(A_22(q),q);
  end proc:
  `MA_22(q) `:=MA_22(q);
  `normalized `:=normal(MA_22(q));
```

$$MA\_22(q) := \frac{1}{(q+3)\,q} - \frac{q+1}{(q+3)^2\,q} - \frac{q+1}{(q+3)\,q^2}$$

$$normalized := -\frac{q^2+2\,q+3}{(q+3)^2\,q^2} \tag{9}$$

There are no critical points!

```
> solve(MA_22(q)=0,q);
```

$$-1 - I\sqrt{2},\ -1 + I\sqrt{2} \tag{10}$$

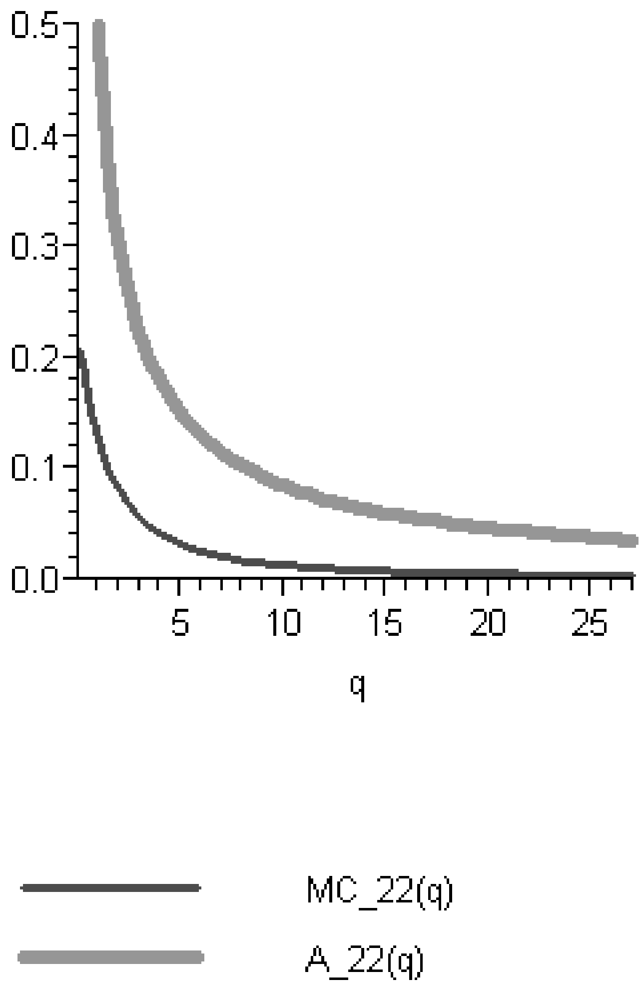
So look at the endpoints!

```
> minimize(A_22(q), q = 0 ..27,location);
```

$$\frac{14}{405},\ \left\{\left[\{q = 27\},\ \frac{14}{405}\right]\right\} \tag{11}$$

Graph A(q) and MC(q) on the same axes

```
> plotMCA:=plot([MC_22(q),A_22(q)],q = 0.1 .. 27,0 .. 0.5,thickness
  = [2,4],legend = [`MC_22(q)`,`A_22(q)`]):
  display(plotMCA);
```



********************Modify Problem to (q^2 +1)/(q+3)*****************

Problem9e.3.4.22a --- Modify C(q):= (q ^2+ 1)/(q + 3)

{\bf Problem 3.4.22} Price \$p\$ and demand \$q\$
are related by the equation \$p = 81 - 3q\$. The total cost of
producing \$q\$ units is \$C(q) = (q ^2+ 1)/(q + 3)\$
\itemitem{(a)} revenue function \$R(q) =?\$
\itemitem{(b)} profit function \$P(q) =?\$
\itemitem{(c)} marginal revenue function \$R'(q) =?\$
\itemitem{(d)} marginal cost \$C'(q) =?\$
\itemitem{(e)} average cost \$A(q) = (C(q)/q) =?\$
\itemitem{(f)} derivative of average cost \$A'(q) =?\$
\itemitem{(g)}Find the \$q\$ for which \$A(q)\$ is minimized. For this \$q\$, compare
the marginal cost \$C'(q)\$ and \$A(q)\$.

```
> p_22a:=proc(q);
     81 - 3*q;
  end proc:
  `p_22a(q) `:=p_22a(q);
  C_22a:= proc(q);
```

```
    (q^2 + 1)/(q + 3);
  end proc:
  `C_22a(q) `:=C_22a(q);
```

$$p\_22a(q) := 81 - 3\,q$$

$$C\_22a(q) := \frac{q^2+1}{q+3} \quad (12)$$

\itemitem{(a)} revenue function $R(q) =?$

```
> R_22a:=proc(q);
    q*p_22a(q);
  end proc:
  `R_22a(q) `:=R_22a(q);
```

$$R\_22a(q) := q\,(81 - 3\,q) \quad (13)$$

\itemitem{(b)} profit function $P(q) =?$

```
> P_22a:=proc(q);
     R_22a(q) - C_22a(q);
  end proc:
  `P_22a(q) `:= P_22a(q);
```

$$P\_22a(q) := q\,(81 - 3\,q) - \frac{q^2+1}{q+3} \quad (14)$$

\itemitem{(c)} marginal revenue function $R'(q) =?$

```
> MR_22a:=proc(q);
    diff(R_22a(q),q);
  end proc:
  `MR_22a(q) `:=MR_22a(q);
```

$$MR\_22a(q) := 81 - 6\,q \quad (15)$$

\itemitem{(d)} marginal cost $C'(q) =?$

```
> MC_22a:=proc(q);
    diff(C_22a(q),q);
  end proc:
  `MC_22a(q) `:=MC_22a(q);`normalized `:=normal(%);
```

$$MC\_22a(q) := \frac{2\,q}{q+3} - \frac{q^2+1}{(q+3)^2}$$

$$normalized := \frac{q^2 + 6\,q - 1}{(q+3)^2} \quad (16)$$

\itemitem{(e)} average cost $A(q) = (C(q)/q) =?$

```
> A_22a:= proc(q);
    C_22a(q)/q;
  end proc:
  `A_22a(q)  `:=A_22a(q);
```

$$A\_22a(q) := \frac{q^2+1}{(q+3)\,q} \tag{17}$$

\itemitem{(f)} derivative of average cost $A'(q) =?$

```
> MA_22a:=proc(q);
    diff(A_22a(q),q);
  end proc:
  `MA_22a(q)  `:=MA_22a(q);
  `normalized  `:=normal(MA_22a(q));
```

$$MA\_22a(q) := \frac{2}{q+3} - \frac{q^2+1}{(q+3)^2\,q} - \frac{q^2+1}{(q+3)\,q^2}$$

$$normalized := \frac{3\,q^2-2\,q-3}{(q+3)^2\,q^2} \tag{18}$$

\itemitem{(g)}Find the q for which $A(q)$ is minimized. For this q, compare the marginal cost $C'(q)$ and $A(q)$.

Select the positive critical number

```
> solve(MA_22a(q)=0,q);
  positive_root:=(%)[1];
  `Minimum Average Cost at `:=positive_root;
  `As a decimal, positive root `:= evalf(positive_root,5);
```

$$\frac{1}{3}+\frac{1}{3}\sqrt{10},\ \frac{1}{3}-\frac{1}{3}\sqrt{10}$$

$$positive\_root := \frac{1}{3}+\frac{1}{3}\sqrt{10}$$

$$Minimum\ Average\ Cost\ at := \frac{1}{3}+\frac{1}{3}\sqrt{10}$$

$$As\ a\ decimal,\ positive\ root := 1.3874 \tag{19}$$

```
> `A_22a(q)   `:=evalf(eval(A_22a(q),q=positive_root),5);
>  `MC_22a(q)  `:=evalf(eval(MC_22a(q),q=positive_root),5);
```

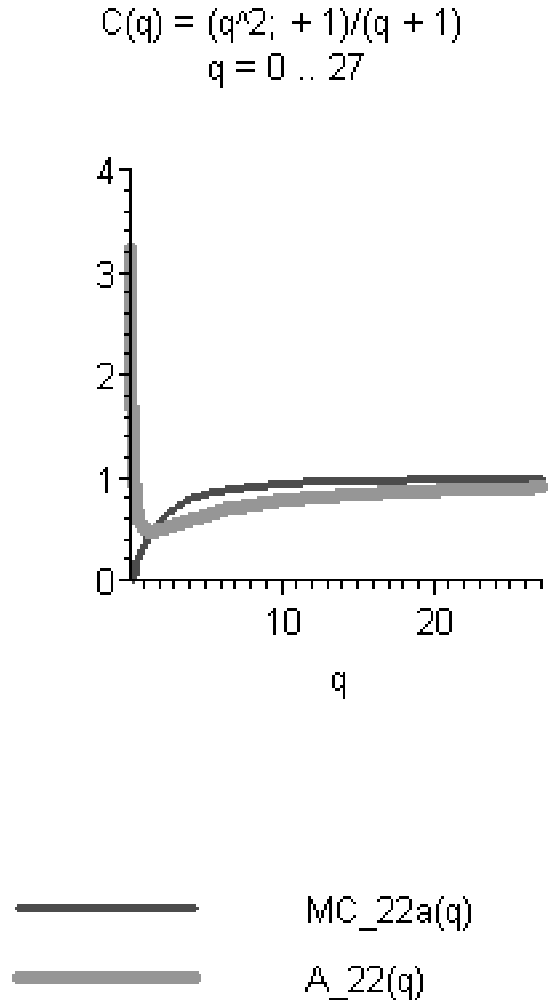
$$A\_22a(q) := 0.48051$$

$$MC\_22a(q) := 0.48049 \tag{20}$$

Graph A(q) and MC(q) on the same axes

```
> plotMCAa:=plot([MC_22a(q),A_22a(q)],q = 0.1 .. 27,0 .. 4,
  thickness = [2,4],legend = [`MC_22a(q)`,`A_22(q)`],
```

```
title=`C(q) = (q^2 + 1)/(q + 1)\nq = 0 .. 27`):
display(plotMCAa);
```



{\bf Problem 3.5.5 variation:} LINEAR PRICE--DEMAND MODEL A store has been selling a popular computer game at the price of \$40 per unit, At this price, players have been buying 50 units per month. The owner of the store wishes to raise the price of the game and estimates that for each \$1 increase in price, 3 fewer units will be sold ({\bf Hint:} $\dfrac{dq}{dp} = \ldots$). If each unit costs the store \$25, at what price p should the game be sold to maximize profit?

\itemitem{(a)}In this problem, express the profit function as a quadratic function of p.

```
> q_5:=proc(p);
     50 -(1/3)*(p - 40);
  end proc:
  `q_5(p) `:=q_5(p);
  C_5:= proc(q);
     25*q;
```

```
end proc:
`C_5(q) `:=C_5(q);`C(p) `:=C_5(q_5(p));
R_5:= proc(p);
 p * q_5(p);
end proc:
`R_5(p) `:=R_5(p);
P_5:=proc(p);
 R_5(p) - C_5(q_5(p));
end proc;
`P_5(p) `:= P_5(p);`normalized `:=normal(P_5(p));
```

$$q\_5(p) := \frac{190}{3} - \frac{1}{3}\,p$$

$$C\_5(q) := 25\,q$$

$$C(p) := \frac{4750}{3} - \frac{25}{3}\,p$$

$$R\_5(p) := p\left(\frac{190}{3} - \frac{1}{3}\,p\right)$$

$$P\_5 := \textbf{proc}(p)\ R\_5(p) - C\_5(q\_5(p))\ \textbf{end proc}$$

$$P\_5(p) := p\left(\frac{190}{3} - \frac{1}{3}\,p\right) - \frac{4750}{3} + \frac{25}{3}\,p$$

$$normalized := \frac{215}{3}\,p - \frac{1}{3}\,p^2 - \frac{4750}{3} \tag{21}$$

\itemitem{(b)} Use differentiation wrt p to find the critical number ($p = \ldots$) of the profit function.

\itemitem{(c)} The graph of the profit function is a parabola. Where is the vertex of the parabola?

```
> P_5_prime:=proc(p);
     diff(P_5(p),p);
  end proc;
  `P_5_prime(p) `:=P_5_prime(p);
  `maximum at p `:= solve(P_5_prime(p)=0,p);
  `vertex at -B/2A `:=(215/3)/(2*(-1/3));
```

$$P\_5\_prime := \textbf{proc}(p)\ diff(P\_5(p), p)\ \textbf{end proc}$$

$$P\_5\_prime(p) := \frac{215}{3} - \frac{2}{3}\,p$$

$$maximum\ at\ p := \frac{215}{2}$$

(22)

$$\textit{vertex at -B/2A} := -\frac{215}{2} \tag{22}$$

Still working on finding a good presentaion of optimization problems {\bf Problem 3.5.10.} There are 320 yards of fencing available to enclose a rectangular field. How should this fencing be used so that the enclosed area is maximized? What is the shape of the {\it optimal field\/}?

If X is the width and Y is the width, then 2X + 2Y = 320.

```
> A_10:=proc(X,Y);
     X * Y;
  end proc;
  `A_10(X,Y) `:= A_10(X,Y);
  Y_10:=proc(X);
   solve(2*X + 2*Y =320,Y);
  end proc;
  `Y_10(X) `:=Y_10(X);
  A_10_X:=proc(X);
     A_10(X,Y_10(X));
  end proc;
  `A_10_X(X) `:=A_10_X(X);
  diff_A_10_X:=proc(X);
     diff(A_10_X(X),X);
  end proc;
  `diff_A_10_X `:=diff_A_10_X(X);
  Crit_10_X:=solve(diff_A_10_X(X)=0,X);
  Crit_10_Y:=Y_10(Crit_10_X);

>
```

$$A\_10 := \textbf{proc}(X, Y)\ Y*X\ \textbf{end proc}$$

$$A\_10(X,Y) := Y X$$

$$Y\_10 := \textbf{proc}(X)\ solve(2*X+2*Y=320, Y)\ \textbf{end proc}$$

$$Y\_10(X) := -X + 160$$

$$A\_10\_X := \textbf{proc}(X)\ A\_10(X, Y\_10(X))\ \textbf{end proc}$$

$$A\_10\_X(X) := (-X + 160)\, X$$

$$diff\_A\_10\_X := \textbf{proc}(X)\ diff(A\_10\_X(X), X)\ \textbf{end proc}$$

$$diff\_A\_10\_X := -2\,X + 160$$

$$Crit\_10\_X := 80$$

$$Crit\_10\_Y := 80 \tag{23}$$

```
> maximize(A_10_X(X),X= 0 .. 320,location);
```

$$6400, \{[\{X=80\}, 6400]\} \quad (24)$$

```
>
```