2009testtwosample.mw
Maple 10 Worksheet for Problems in Math 165 - Calculus for Business.
N.B. These problems are a lot easier to do "by hand"
First load plots and student:

N.B. A Maple command such as eval(f(x),x=2) is the instruction
``Evaluate f(2)" or
``evaluate the function f(x) at x = 2."
a:= b assigns the value 'b' to the name 'a'
Use a semicolon ; to execute a 'Maple Command' and display the output.
Using a colon : executes a command without displaying the output.
I (JL) usually write functions as a procedure such as the "x square funtion:
square_function:= proc(x);x^2 ; end proc;
Other ways to write functions:
square_function:= x -> x^2;
`%` is the last computed expression (Simlar to ANS on your calculator).

1. Find the intervals of increase and decrease for
14* x^3 + 0*x^2 - 1512*x -4
Find dy/dx and solve > 0, < 0

$$f\_1(x) := 14\,x^3 - 4 - 1512\,x$$

$$deriv\_f\_1(x) := 42\,x^2 - 1512$$

$$critical\_nos := 6, -6$$

$$int\_INC := RealRange(-\infty, Open(-6)), RealRange(Open(6), \infty)$$

$$int\_DEC := RealRange(Open(-6), Open(6))$$

2. Maximmize R_2(x) = (24*x - x^2)/(x^2 + 24)

$$R\_2(x) := \frac{24\,x - x^2}{x^2 + 24}$$

$$deriv\_R\_2(x) := \frac{24 - 2\,x}{x^2 + 24} - \frac{2\,(24\,x - x^2)\,x}{(x^2 + 24)^2}$$

$$simplified := -\frac{24\,(x^2 - 24 + 2\,x)}{(x^2 + 24)^2}$$

$$crit\_no := -6, 4$$

$$answer\_2 := x = 4, R\_2(4) = 2$$

$$OR := [4, 2]$$

3. Critical numbers and classify 1/(x^2 - 8*x + 7)

$$f\_3(x) := \frac{1}{x^2 - 8\,x + 7}$$

$$deriv\_f\_3 := -\frac{2\,x - 8}{(x^2 - 8\,x + 7)^2}$$

$$vertical\_asymptotes := 7, 1$$

$$crit\_nos := 4$$

answer_3 := x=4 is a rlative maximum - look at the graph

4.

$$C := x \rightarrow 0.1\ RealDomain{:}\text{-}{\wedge}(x, 2) + 20\,x + 500$$

$$A := x \rightarrow \frac{C(x)}{x}$$

$$minimum\_average\_at := \{x = 70.71067812\} \qquad (1)$$

5. find max min neither x^4 - 4* x^2 + 1

$$f\_5(x) := x^4 - 4\,x^2 + 1$$

$$deriv\_f\_5(x) := 4\,x^3 - 8\,x$$

$$deriv2\_f\_5(x) := 12\,x^2 - 8$$

$$crit\_nos := \left[0, \sqrt{2}, -\sqrt{2}\right]$$

$$deriv\_2\_at\_crit\_nos := [-8, 16, 16]$$

answer_5 := rel MAX at x = 0, rel MIN at \pm sqrt(2)

6.   x^3 +3*x^2 - 9*x +1 concave down

$$f\_6(x) := x^3 - 3\,x^2 - 9\,x + 1$$

$$deriv\_f\_6(x) := 3\,x^2 - 6\,x - 9$$

$$deriv2\_f\_6(x) := 6\,x - 6$$

$$answer\_6 \text{ - concave down on } := RealRange(-\infty, Open(1))$$

7 inflection x^4 + 6*x^3 - 24* x^2 + 26; solve y" =0. actually one should check that sign y" changes - by factoring in this case

$$f\_7(x) := x^4 + 6\,x^3 - 24\,x^2 + 26$$

$$deriv\_f\_7(x) := 4\,x^3 + 18\,x^2 - 48\,x$$

$$deriv2\_f\_7(x) := 12\,x^2 + 36\,x - 48$$

$$possible\_inflection := 1, -4$$

$$concave\_up := RealRange(-\infty, Open(-4)), RealRange(Open(1), \infty)$$

$$concave\_down := RealRange(Open(-4), Open(1))$$

$$answer\_7 := [[1, 9], [-4, -486]]$$

8. Critical number x^4 + 6*x^3 - 24* x^2 + 26

$$f\_8(x) := 2\,x^2 - 8\,x + 7$$

$$deriv\_f\_8(x) := 4\,x - 8$$

$$deriv2\_f\_8(x) := 4$$

$$answer\_8 := 2 \qquad (2)$$

9. Analysis of derivative of x^4 + 6*x^3 - 24* x^2 + 26

$$f\_9(x) := -x^3 + 12\,x^2 + 144\,x + 55$$

$$deriv\_f\_9(x) := -3\,x^2 + 24\,x + 144$$

$$deriv2\_f\_9(x) := -6\,x + 24$$

$$answer\_max\_rate := 192, \{[\{t = 4\}, 192]\}$$

$$answer\_min\_rate := 144, \{[\{t = 0\}, 144]\}$$

$$answer\_max\_rate\_of\_rate := 24, \{[\{t = 0\}, 24]\}$$

$$OR := 24, \{[\{x = 0\}, 24]\}$$

$$total\_answer\_9 := [t = 4, t = 0, t = 0] \qquad (3)$$

10. asymptotes 1/(x - 2)

$$f\_10(t) := \frac{1}{t-2}$$

$$first\_deriv := -\frac{1}{(t-2)^2}$$

$$second\_deriv := \frac{2}{(t-2)^3}$$

$$vertical\_asymptote := \{x = 2\}$$

$$horizontal\_asymptote := y = 0 \qquad (4)$$

11 abs max t^5 - t^4 on [-1,1]

$$f\_11(x) := x^5 - x^4$$

$$first\_deriv := 5\,t^4 - 4\,t^3$$

$$f\_11(t) := t^5 - t^4$$

$$critical\_numbers := 1, 0, 0, 0, 0$$

$$endpoints := \{-1, 1\}$$

$$where\_to\_look := \{-1, 0, 1\}$$

$$critical\_points := \{[0, 0], [1, 0], [-1, -2]\}$$

answer_11 := abs max value = 0, occurs at t= 0,1

12.WRONG ANSWER with p = 44-x; Answer OK for p = (45 - x)-- see below

$$p\_12(x) := 44 - x$$

$$R\_12(x) := x\,(44 - x)$$

$$C\_12(x) := x^2 + 5\,x + 6$$

$$P\_12(x) := 39\,x - 2\,x^2 - 6$$

$$MP\_12(x) := 39 - 4\,x$$

$$answer\_12 := \left\{x = \frac{39}{4}\right\} \qquad (5)$$

12 again with the function p changed to p = 45-x

$$p\_12m(x) := 45 - x$$

$$R\_12m(x) := x\,(45 - x)$$

$$C\_12m(x) := x^2 + 5\,x + 6$$

$$P\_12m(x) := 40\,x - 2\,x^2 - 6$$

$$MP\_12m(x) := 40 - 4\,x$$

$$answer\_12m := \{x = 10\} \tag{6}$$

13. max profit

$$q\text{-}13(p) := \frac{1100}{3} - \frac{1}{3}\,p$$

$$P\_13(p) := (p - 40)\left(\frac{1100}{3} - \frac{1}{3}\,p\right)$$

$$deriv\_P\_13(p) := 380 - \frac{2}{3}\,p$$

$$max\_profit\_1t := \{p = 570\} \tag{7}$$

14. max revenue train fare

$$q\_14(p) := 1600.000000 - 1000.000000\,p$$

$$R\_14(p) := p\,(1600.000000 - 1000.000000\,p)$$

$$deriv\_R\_14(p) := 1600.000000 - 2000.000000\,p$$

$$max\_profit\_at := \{p = 0.8000000000\} \tag{8}$$

15. x>0, y > 0, x+y = 10, maximize xy^2

$$constraint := x + y = 10$$

$$M\_15\_2(x,y) := x\,y^2$$

$$solve\_for\,y := -x + 10$$

$$M\_15(x) := x\,(x - 10)^2$$

$$deriv\_M\_15(x) := (x - 10)^2 + 2\,x\,(x - 10)$$

$$x\_answer := \left\{x = \frac{10}{3}\right\}$$

$$y\_answer := \frac{20}{3} \tag{9}$$

16 maximize average cost; C = (1/8) * x^2 +4*x +200

$$C\_16(x) := \frac{1}{8}\,x^2 + 4\,x + 200$$

$$A\_16(x) := \frac{\frac{1}{8}\,x^2 + 4\,x + 200}{x}$$

$$deriv\_A\_16(x) := \frac{\frac{1}{4}x + 4}{x} - \frac{\frac{1}{8}x^2 + 4x + 200}{x^2}$$

$$OR := \frac{1}{8}\,\frac{x^2 - 1600}{x^2}$$

$$answer\_16 := \{x = 40\} \qquad (10)$$

17.Wrong Answer for q = (300 - p^2)/60. Answer OK for q = (30000 - p^2)/60.-- see below

$$q\_17(p) := \; := 5 - \frac{1}{60}p^2$$

$$deriv\_q\_17(p) := -\frac{1}{30}p$$

$$E\_17(p) := \frac{2p^2}{-300 + p^2}$$

$$demand\_of\_unit\_elasticity := \{p = 10\}$$

$$answer\_17\ elastic\_demand := p > 10 \qquad (11)$$

17.again with q =. (30000 - p^2)/60 works -- see below

$$q\_17(p) := \; := 500 - \frac{1}{60}p^2$$

$$deriv\_q\_17(p) := -\frac{1}{30}p$$

$$E\_17(p) := \frac{2p^2}{-30000 + p^2}$$

$$demand\_of\_unit\_elasticity := \{p = 100\}$$

$$answer\_17\ elastic\_demand := p > 100 \qquad (12)$$

18 Variation of Price - Demand Yield = Price

$$Y\_18(x) := 280 - 2x$$

$$Y\_18\ (x) := 280 - 2x$$

$$answer\_18 := 9800,\ \{[\{x = 70\}, 9800]\} \qquad (13)$$

19. 0 < x < 10, x+y =10, maximize x^2*y^2

$$constraint := x + y = 10$$

$$M\_19\_2(x,y) := x^2 y^2$$

$$solve\_for\ y := -x + 10$$

$$M\_19(x) := x^2 (x - 10)^2$$

$$deriv\_M\_19(x) := 2x(x - 10)^2 + 2x^2(x - 10)$$

$$x\_answer := \{x = 5\}$$

$$y\_answer := 5 \qquad (14)$$

20.future value

$$I\_20 := 3000$$

$$r\_20 := 0.10$$

$$T\_20 := 9$$

$$answer\_20 := 7378.809333 \qquad (15)$$

21 Wrong Answer for f(1) = 100, multiply by 100 every year. f(3) = 10^6. Answer OK forh f(1) = 10.

$$f\_21(1) := 100$$

$$Answer\_21\ f\_21(3) = f\_21(1))\text{^}3 := 1000000$$

$$f\_21(1) := 10$$

$$Answer\_21\ f\_21(3) = f\_21(1))\text{^}3 := 1000 \qquad (16)$$

22. future value

$$PV\_22 := 4000$$

$$r\_22 := 0.08$$

$$T\_22 := 11$$

$$answer\_22 := 2.410899706\ I\_22 \qquad (17)$$

23. count :-- half life is six weeks

$$F\_23(x) := \left(\frac{1}{2}\right)^{\frac{1}{6}x}$$

$$answer\_23 := \frac{1}{8} \qquad (18)$$

24. count: every 100 years multiply by 600/800

$$F\_24(t) := 800\left(\frac{3}{4}\right)^{\frac{1}{100}t}$$

$$answer\_24 := 253.1250 \qquad (19)$$

25. solve 2*ln(x) - (1/3)*ln(x^2):=4

$$EQN := 2\ln(x) - \frac{1}{3}\ln\left(x^2\right) = 4$$

$$use\ ln\ rules\ on\ both\ sides := \left[\frac{4}{3}\ln(x), 4\right]$$

$$take\ exp\ of\ both\ sides := \left[x^{4/3}, \mathrm{e}^4\right]$$

$$take\ 3/4\ power\ of\ both\ sides := \left[x, e^3\right]$$

$$answer\_25 := e^3 \qquad (20)$$

26. carbon dating

$$Age\_(t) := A\_0\left(\frac{1}{2}\right)^{\frac{1}{5730}t}$$

(21)

$$Answer\_26 := 16086.14370 \quad (21)$$

27. 5^*x* = *e*^8`; `*note* ln(*e*) = 1; *take* ln *of both sides* (*easier* **by** *hand*!

$$EQN := 5^x = e^8$$

$$apply\ ln\ to\ both\ sides := [x \ln(5), 8]$$

$$answer\_27 := \frac{8}{\ln(5)} \quad (22)$$

28. Find the interest rate solve({PV_28* exp(r*T_28) = FV_28},r)

$$FV\_28 := 6000$$

$$PV\_28 := 3000$$

$$T\_28 := 14.$$

$$answer\_28 := \{r = 0.04951051290\}$$

$$ln(2)/(14) := 0.04951051290 \quad (23)$$