Maple 10 Worksheet for Problems in Math 165 - Calculus for Business.
Answer Key for exam02sample

First load plots and student:

```maple
> restart: with( student ):with (plots):with(plottools):
```

**N.B.** A Maple command such as eval(f(x),x=2) is the instruction
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Problem 2
Solve \( f' = 0 \) to find all critical numbers

\[
\begin{align*}
> \ f_2 & := \text{proc}(x); \\
& \quad \quad 2x^2 - 8x + 7; \\
& \quad \quad \text{end proc;} \ f_2(x) := f_2(x) ; \\
> \ d_{dx} f_2(x) & := \text{diff}(f_2(x),x); \\
> \ crit\_num & := \text{solve}(x = 0,x); \\
\ & f_2(x) := 2x^2 - 8x + 7 \\
\ & d_{dx} f_2(x) := 4x - 8 \\
\ & crit\_num := 2
\end{align*}
\]

(3)

Problem 3
Solve \( f' = 0 \) to find critical numbers, check the sign of \( f' \) in between.

\[
\begin{align*}
> \ f_3 & := \text{proc}(x); \\
& \quad \quad 4x^3 + 18x^2 - 120x - 4; \\
& \quad \quad \text{end proc;} \ f_3(x) := f_3(x) ;
\end{align*}
\]
\[ f_3(x) := 4x^3 + 18x^2 - 120x - 4 \]
\[ d_{dx}f_3(x) := 12x^2 + 36x - 120 \]
\[ \text{factored} := 12(x+5)(x-2) \]

\[ \text{int}_\text{increase} := \text{RealRange}(-\infty, \text{Open}(-5)), \text{RealRange}(\text{Open}(2), \infty) \]

\[ \text{int}_\text{decrease} := \text{RealRange}(\text{Open}(-5), \text{Open}(2)) \]
end proc:  `f_4(x)` := f_4(x);
f_4_prime:=proc(x);
  diff(f_4(x),x);
end proc;  `f_4_prime(x)` := f_4_prime(x);
crit_numbers:= solve(f_4_prime(x),x);
  f_4(x) := 4 x^2 - 6 x + 1
  f_4_prime := proc(x) diff(f_4(x), x) end proc
  f_4_prime(x) := 8 x - 6
  crit_numbers := \frac{3}{4}

Problem 5
Check for vertical asymptotes!
> f_5:=proc(x);
  5/(x^2 - 8*x + 12);
end proc:  `f_5(x)` := f_5(x);
`vertical asymptote at x` := solve(x^2 - 8*x + 12 = 0, x);
Vertical asymptotes at x = 2, 6.
  f_5(x) := \frac{5}{x^2 - 8 x + 12}
  vertical asymptote at x := 6, 2

Now solve f' = 0
> d_dx_f_5(x):= diff(f_5(x), x);
> crit_num:=solve(d_dx_f_4(x) =0, x);
At x = 4, d_dx_f_4 changes from - to + so LOCAL MINIMUM
  d_dx_f_5(x) := \frac{-5 (2 x-8)}{(x^2-8 x+12)^2}
  crit_num := RootOf(d_dx_f_4(_Z))

A graph for problem 5
> plot_5:=plot(f_5(x),x = - 10 .. 10, -10 ..10, thickness = 2);
  display(plot_5);
Problem 6 ANSWER KEY MISTAKE
Profit = Revenue - Cost
Revenue(x) = x * (49 - x)
> R_6 := proc(x);
> x*(49 - x) - (x^2 + 4*x + 7);
end proc:`R_6(x)` := R_6(x);
> MP_6(x) := diff(R_6(x),x);
> max_profit_x := solve(MP_6(x)=0, x);

\[ R_6(x) := x(49-x) - x^2 - 4x - 7 \]
\[ MP_6(x) := 45 - 4x \]
\[ max_profit_x := \frac{45}{4} \]  

(7)

Problem 7
Solve f' = 0. f' is a quadratic
Check sign of f''.
At a critical point, f'' > 0 says local maximum, f'' < 0 says local maximum.
> f_8(x) := 2*x^3 - 3 * x^2 - 12 * x +13;
d_dx_f_8(x) := diff(f_8(x),x);
crit_num:=solve(d_dx_f_8(x) = 0 , x);
d2_dx2_f_8(x):= diff(f_8(x),x$2);
f_prime_prime_crit_num:=
    [eval(d2_dx2_f_8(x),x= crit_num[1]),eval(d2_dx2_f_8(x),x= crit_num[2])];
f_8(x) := 2 x^3 - 3 x^2 - 12 x + 13
d_dx_f_8(x) := 6 x^2 - 6 x - 12
crit_num := 2, -1
d2_dx2_f_8(x) := 12 x - 6
f_prime_prime_crit_num := [18, -18] (8)

********************************************************************************
Problem 8
Note: q = q(p) is a function of price.
N.B. Statement "14 more buzzers for every 12 cent decrease in price means
dq/dp = 14/(-0.12), and "price demand of elasticity" is
E(p) = (p/q)*(dq/dp)
Elasticity:= (1.40/60)*(14/(-0.12));
Elasticity := -2.722222223 (9)

> q_8:=proc(p); 60 + (14/(-0.12))*(p - 1.40);end proc:
`q_8(x)` := q_8(x);

> P_8(p):=(p - .40)* q_8(p):`P_8(p)` :=P_8(p);
> d_dp_P_8(p):= diff(P_8(p),p);
> max_profit_price:=solve(d_dp_P_8(p), p);
    q_8(x) := 223.3333334 - 116.6666667 x
    P_8(p) := (p - 0.40) (223.3333334 - 116.6666667 p)
    d_dp_P_8(p) := 270.0000001 - 233.3333334 p
    max_profit_price := 1.157142857

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Problem 9
Note Cost is a function of p
Profit(p) = p*(25 - p) - C(p)
> D_9(p) := 28 - 5 * p;
    P_9(p):=p* D_9(p) - (p^2 + 4*p);
    d_dp_P_9(p):= diff( P_9(p),p);
    crit_num:=solve(d_dp_P_9(p),p);
    max_profit_P:=eval(P_9(p),p=crit_num);
evalb(max_profit_P = 24);
    D_9(p) := 28 - 5 \ p
\[ P_9(p) := p (28 - 5 p) - p^2 - 4 p \]
\[ d/dp P_9(p) := 24 - 12 p \]
\[ \text{crit num} := 2 \]
\[ \text{max profit } P := 24 \]
\[ \text{true} \]  

Problem (10): Note 0 \leq n \leq 10. Check the Endpoints!
Hiring the first 5 employees decreases net revenue!

\[ R_{10} := \text{proc}(n) : \]
\[ - 3n^4 + 40n^3 - 126n^2 + 15; \]
\[ \text{end proc; } \]
\[ d/dn R_{10}(n) := \text{diff}(R_{10}(n),n); \]
\[ \text{crit num} := \text{solve}(d/dn R_{10}(n) = 0 , n); \]
\[ \text{values at crit end} := [\text{eval}(R_{10}(n),n = \text{crit num}[1]), \]
\[ \text{eval}(R_{10}(n),n = \text{crit num}[2]), \]
\[ \text{eval}(R_{10}(n),n = \text{crit num}[3]), \]
\[ \text{eval}(R_{10}(n),n = 10)] \]  

\[ R_{10}(n) := -3 n^4 + 40 n^3 - 126 n^2 + 15 \]
\[ d/dn R_{10}(n) := -12 n^3 + 120 n^2 - 252 n \]
\[ \text{crit num} := 0, 7, 3 \]
\[ \text{values at crit end} := [15, 358, -282, -2585] \]  

Maximum 358 is at n = 7
A Graph for Problem 10

\[ \text{plot}(R_{10}(n), n = 0 .. 10, \text{thickness} = 2); \]
Problem 11
\[ P_{10} := 19000 \cdot \exp(-.06 \cdot 10) \]
\[ P_{10} := 19000 \cdot \exp(-.06 \cdot 10); \]
\[ P_{10} := 10427.42109 \] \hfill (13)

Problem 12 \( f(t) := f(0) \cdot (\frac{f(1)}{f(0)})^t \)
\[ f_{12}(t) := 1 \cdot (7/1)^t; \]
\[ f(3) := \text{eval}(f_{12}(t), t=3); \]
\[ f_{12}(t) := 7^t \]
\[ f(3) := 343 \] \hfill (14)

Problem (13) \( P(0) := P(t) \cdot \exp(-r \cdot t) \)
\[ P_{10} := 5000 \cdot \exp(-.07 \cdot 10); \]
\[ \text{evalb}(|P_{10} - 2000| < 1); \]
\[ P_{10} := 2482.926519 \]
\[ \text{false} \] \hfill (15)