Name: _____

MthT 430 Final Assessment 2001

Definitions

1. Define $(\epsilon - \delta)$: $\lim_{x \to a} f(x) = L$.

2. Define: $\lim_{x \to a^-} f(x) = L$.

3. Define: The function f is continuous at a.

4. Define: The set of numbers A is bounded above.

5. Define: The number b is the *least upper bound* of a set of numbers A.

6. Give an example of two functions f and g such that $f \circ g = g \circ f$. Be sure to verify that the domains are the same.

7. Give an example of a bounded function f defined for all real numbers such that $\lim_{x\to 0} f(x)$ does not exist.

8. Give an example of a bounded set of numbers A which has a greatest element. Give the least upper bound of this set A.

9. Give an example of a nonempty bounded set of numbers A which has no greatest element. Give the least upper bound of this set A.

10. Give an example of a nonempty bounded set A_Q of rational numbers whose least upper bound is not a rational number.

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Proofs

- 11. Let f be defined on [0,1) be such that
 - f is increasing on [0,1) (If $0 \le x_1 < x_2 < 1$, then $f(x_1) < f(x_2)$.)
 - f is bounded above on [0, 1).

Prove that

$$\lim_{x \to 1^{-}} f(x) = L$$

exists.

Qualitative Properties of Functions

12. Water drips very slowly into a circular bottle (beaker, flask) so that the graph of the Height (in cm) as a function of Volume (in cm³) is shown below.

Figure-1



Draw a side view of the bottle. Carefully explain as many features as you can about the shape of the bottle and explain how they are related to the Height–Volume graph.

Essay

13. (Letter Grade: A - E) In the exam booklet, write an essay on a topic of your choice that is very relevant to the material considered in the course. Your essay should include at least one substantial example and at least one substantial theorem and its proof.