

**MthT 430 Midterm Assessment 2002****I. Definitions**

1. (10 points) Define  $(\epsilon\text{-}\delta)$ :  $\lim_{x \rightarrow a} f(x) = L$ .
2. (10 points) Is the following a correct definition of limit? Explain your answer.

**Definition.**

$$\lim_{x \rightarrow a} f(x) = L$$

means: For some  $\epsilon > 0$ , there is a  $\delta > 0$ , such that, for all  $x$ , if  $0 < |x - a| < \delta$ , then  $|f(x) - L| < \epsilon$ .

**II. Examples**

3. (10 points) Give an example of two functions  $f$  and  $g$  such that  $f \circ g = g \circ f$ . Be sure to verify that the domains are the same.
4. (10 points) Give an example of a function  $f(x)$  defined for all real numbers such that  $\lim_{x \rightarrow 0} f(x)$  exists but does not equal  $f(0)$ .
5. (20 points) Let

$$F(x) = \sqrt{x^2 - 1},$$
$$G(x) = \sqrt{1 - x^2}.$$

Describe:

- domain  $(F)$  and domain  $(G)$ .
- domain  $(F + G)$
- domain  $(G \circ F)$
- domain  $\left(\frac{F}{G}\right)$
- domain  $(F \circ G)$

**III. Proofs**

6. (20 points) Show, using only P1 – P9:

For all  $a$ ,  $a \cdot 0 = 0$ .

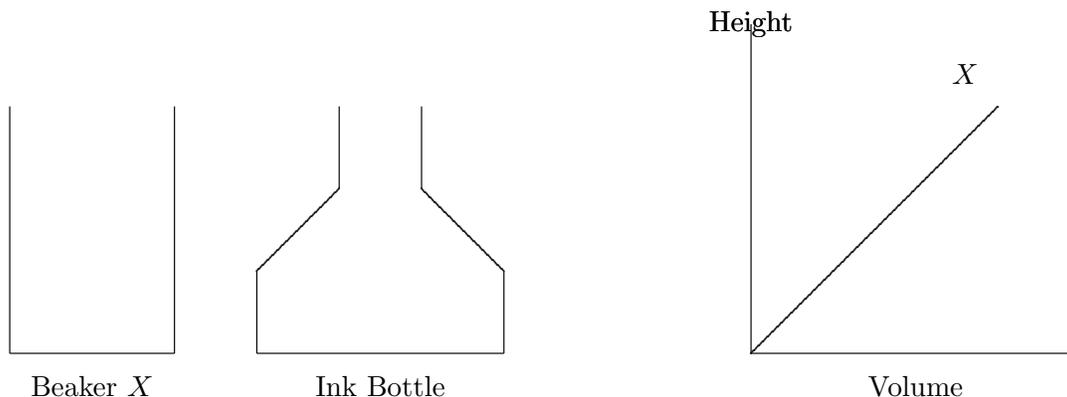
You may abbreviate (*distributive, ...*).

7. (20 points) Show by mathematical induction or otherwise: (Bernoulli's Inequality) For all natural numbers  $n = 1, 2, \dots$ , for  $x > -1$ ,

$$(1 + x)^n \geq 1 + nx.$$

**IV. Qualitative Properties of Functions**

8. (30 points) The graph below shows how the height of a liquid in beaker  $X$  varies as water is steadily dripped into it. Copy the graph, and *on the same diagram* show the height–volume relationship for the Ink Bottle.



Describe the features of the graph you have drawn. Your description should include

- The domain of the function
- The intervals of monotonicity (Increasing, Decreasing)
- The intervals of constant concavity and/or linearity
- Other observations ...

A person reading your description of the graph should be able to reproduce the graph of the function (and if she's good, guess that it came from something shaped like the ink bottle!).

**V. Essay**

9. (Letter Grade: A - E) In the exam booklet, write an essay on a topic of your choice that is very relevant to the material considered in the course. Your essay should include at least one substantial example and at least one substantial theorem and its proof.