21. Assume that if

\[ |x - x_0| < \min\left(\frac{\epsilon}{2(|y_0| + 1)}, 1\right), \]
\[ |y - y_0| < \frac{\epsilon}{2(|x_0| + 1)}. \]

then

\[ |xy - x_0y_0| < \epsilon. \]

The trick is to write \( xy - x_0y_0 \) in terms of \( x - x_0 \) and \( y - y_0 \). There are several ways to do this, but the one which works best is

\[
xy - x_0y_0 = x(y - y_0 + y_0) - x_0y_0
= x(y - y_0) + (x - x_0)y_0
= I + II.
\]

Then

\[
|I| \leq |x| |y - y_0|
\leq (|x_0| + |x - x_0|) |y - y_0|
\leq (|x_0| + 1) |y - y_0|
< \frac{\epsilon}{2},
\]
\[
|II| \leq |x - x_0| |y_0|
< \frac{\epsilon}{2}.
\]

Thus

\[
|xy - x_0y_0| \leq |I| + |II|
< \frac{\epsilon}{2} + \frac{\epsilon}{2}
= \epsilon.
\]