MthT 430 Projects Chapter 1a

page 2 In class September 5, 2007

The Triangle Inequality and Applications

For the time being, assume (P1) – (P12), and

$$|a| = \begin{cases} a, & a \geq 0 \\ -a, & a \leq 0. \end{cases}$$

The *Triangle Inequality* says that

$$|a+b| \leq |a| + |b| .$$

1. Show that

$$|a-b| \leq |a| + |b| .$$

2. Show that

$$|ab| = |a| \cdot |b| .$$

3. Show that

$$|a| \leq |a-b| + |b| .$$

4. Show that

$$||a| - |b|| \leq |a-b| .$$

5. List *all* numbers such that $|a| = 0$.

6. Show that if

$$0 < a < b,$$

then

$$0 < b^{-1} < a^{-1}.$$

Work on Chapter 1, Problems 20 and 21 in Spivak

20. Prove that if

$$|x - x_0| < \epsilon/2 \text{ and } |y - y_0| < \epsilon/2,$$

then

$$|(x+y) - (x_0 + y_0)| < \epsilon.$$

Œ

Let the set of numbers Œ consist of the two objects

$$\{\text{odd}, \text{even}\}$$

Here is the addition table:

+ (plus)	odd	even
odd	even	odd
even	odd	even

Here is the multiplication table:

· (times)	odd	even
odd	odd	even
even	even	even

This set of **Numbers** satisfies (P1) – (P9).

1. Which element has the role of 0?

2. Which element has the role of 1?

3. Is it possible to define a set of positive numbers P such that (P10) – (P12) are satisfied?

4. Is it possible to define an *absolute value* on Œ with all of the properties:

A_1 For all a in Œ, $|a|$ is a real number, $|a| \geq 0$,

A_2 $|a| = 0$ iff $a = 0$,

A_3 $|a + b| \leq |a| + |b|$,

A_4 $|a \cdot b| = |a| \cdot |b|$?