MthT 430 Chapter 2b Projects

In class September 12, 2007, Turn in September 19, 2007

Rational and Irrational Numbers

1. Prove that $\sqrt{3}$ is irrational.

2. Let the set of numbers $\mathbf{Q}_{\sqrt{3}}$ consist all the real numbers, x, of the form

$$x = p + q\sqrt{3},$$

where p and q are rational numbers. .

- Prove that if $x = p + q\sqrt{3}$, where p and q are rational numbers, then

$$\begin{aligned} x^{-1} &= \frac{1}{p + q\sqrt{3}} \\ &= a + b\sqrt{3}, \end{aligned}$$

for some rational a and b.

- Prove that if $x = p + q\sqrt{3}$ where p and q are rational numbers, and m is a natural number, then $x^m = a + b\sqrt{3}$ for some rational a and b.

Remark: One can show that $\mathbf{Q}_{\sqrt{3}}$ satisfies P1 – P12; more briefly: $\mathbf{Q}_{\sqrt{3}}$ is an ordered field.

Cauchy – Schwartz Inequality

3. Prove by mathematical induction or otherwise:

$$\left(\sum_{j=1}^{m} x_j \, y_j \right)^2 \leq \left(\sum_{j=1}^{m} x_j^2 \right) \cdot \left(\sum_{j=1}^{m} y_j^2 \right).$$