# MthT 430 Notes Chapter 3a Functions

See the discussion and examples in Chapter 3 of Spivak.

**Provisional Definition.** *A function is a rule which assigns, to each of certain real numbers, some other real number.*

The set of *certain real numbers* (indicating numbers) is called the *domain* of the function.

Unless the domain is explicitly restricted further, the convention is that the domain of the function is understood to consist of all the numbers for which the definition makes any sense at all.

## Constructing Functions from Others

Pay attention to the domain.

- Composition
$$f \circ g : (f \circ g)(x) = f(g(x)),$$
domain $f \circ g$: $x \in$ domain $g$ and $g(x) \in$ domain $f$.

- Sums
$$f + g : (f + g)(x) = f(x) + g(x),$$
domain $(f + g)$: $x \in$ domain $f$ and $x \in$ domain $g$.

- Products
$$f \cdot g : (f \cdot g)(x) = f(x) \cdot g(x),$$
domain $(f \cdot g)$: $x \in$ domain $f$ and $x \in$ domain $g$.

- Quotients ($g \not\equiv 0$ on domain $g$)
$$\frac{f}{g} : \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)},$$
domain $\left(\frac{f}{g}\right)$: $x \in$ domain $f$ and $x \in$ domain $g$ and $g(x) \neq 0$.

**Definition.** *A* **function** *is a collection of pairs of numbers with the following property: If $(a, b)$ and $(a, c)$ are both in the collection, then $b = c$.*

**Definition.** *If $f$ is a function, the* **domain** *of $f$ is the set of all $a$ for which there is some $b$ such that $(a, b)$ is in $f$. If $a$ is in the domain of $f$, then there is a* unique *number $b$ such that $(a, b)$ is in $f$. This unique $b$ is denoted by $f(a)$.*