MthT 430 Projects Chapter 5b Limits – Equivalent Definitions (sic)

Limits – Definitions (sic)

Definition. *(Actual, p. 96)*

$$\lim_{x \to a} f(x) = L$$

means: For every $\epsilon > 0$, there is some $\delta > 0$ such that, for all x, if $0 < |x - a| < \delta$, then $|f(x) - L| < \epsilon$.

Some Correct and Incorrect Variations

Decide which, if any, of the Definitions A–N are equivalent to the *actual* definition of

$$\lim_{x \to a} f(x) = L.$$

Definition A.

$$\lim_{x \to a} f(x) = L$$

means: For every $\epsilon > 0$, there is some $\delta > 0$ such that, for all x, if $0 < |x - a| < \delta$, then $|f(x) - L| < \epsilon$.

Definition B.

$$\lim_{x \to a} f(x) = L$$

means: For every $\epsilon > 0$, there is some $\delta > 0$ such that for some x, if $0 < |x - a| < \delta$, then $|f(x) - L| < \epsilon$.

Definition C.

$$\lim_{x \to a} f(x) = L$$

means: For every $\epsilon > 0$, there is some $\delta > 0$ such that for some x, $0 < |x - a| < \delta$, $|f(x) - L| < \epsilon$.

Definition D.

$$\lim_{x \to a} f(x) = L$$

means: For every $\epsilon > 0$, there is some $\delta > 0$ such that, for all x, if $|x - a| < \delta$, then $|f(x) - L| < \epsilon$.

Definition E.

$$\lim_{x \to a} f(x) = L$$

means: For every $\epsilon > 0$, there is some $\delta > 0$ such that, for all x, if $|f(x) - L| < \epsilon$, then $0 < |x - a| < \delta$.

Definition F.

$$\lim_{x\to a} f(x) = L$$

means: For every $\epsilon > 0$*, there is some* $\delta > 0$ *such that* $|f(x) - L| < \epsilon$*, and* $0 < |x - a| < \delta$*.*

Definition G.

$$\lim_{x\to a} f(x) = L$$

means: For every $\delta > 0$*, there is some* $\epsilon > 0$ *such that, for all* x*, if* $0 < |x - a| < \epsilon$*, then* $|f(x) - L| < \delta$*.*

Definition H.

$$\lim_{x\to a} f(x) = L$$

means: For every $\epsilon > 0$*, there is some* $\delta > 0$ *such that, for all* x*, if* $0 < |x - a| < \epsilon$*, then* $|f(x) - L| < \delta$*.*

Definition I.

$$\lim_{x\to a} f(x) = L$$

means: For every $\epsilon > 0$, $\delta > 0$, $0 < |x - a| < \delta$, $|f(x) - L| < \epsilon$*.*

Definition J.

$$\lim_{x\to a} f(x) = L$$

means: For every $\epsilon > 0$*, there is some* $\delta > 0$ *such that, for all* x*, if* $0 < |x - a| < \delta$*, then* $|f(x) - L| < 20\epsilon$*.*

Definition K.

$$\lim_{x\to a} f(x) = L$$

means: For every $\epsilon > 0$*, there is some* $\delta > 0$ *such that, for all* x*, if* $0 < |x - a| < 20\delta$*, then* $|f(x) - L| < \epsilon$*.*

Definition L.

$$\lim_{x\to a} f(x) = L$$

means: For every $\epsilon > 0$*, there is some* $\delta > 0$ *such that, for all* x*, if* $0 < |x - a| < 10^{-6}\delta$*, then* $|f(x) - L| < 10^{6}\epsilon$*.*

Definition M.

$$\lim_{x\to a} f(x) = L$$

means: There is a number M *such that for every* $\epsilon > 0$*, there is some* $\delta > 0$ *such that, for all* x*, if* $0 < |x - a| < 20\delta$*, then* $|f(x) - L| < M\epsilon$*.*

Definition N.

$$\lim_{x \to a} f(x) = L$$

means: There is a number M such that for every $\epsilon > 0$, there is some $\delta > 0$ such that, for all x, if $0 < |x - a| < \delta/M$, then $|f(x) - L| < M\epsilon$.

Definition 10^6.

$$\lim_{x \to a} f(x) = L$$

means: For every $\epsilon > 0$, there is some $\delta > 0$ such that, for all x, if $0 < |x - a| < 10^6\delta$, then $|f(x) - L| < \epsilon$.

$10^6 \Rightarrow$ **Actual**: Fix $\epsilon > 0$. By 10^6, there is an $\delta > 0$ such that, for all x, if $0 < |x - a| < 10^6\delta$, then $|f(x) - L| < \epsilon$. If $0 < |x - a| < \delta$, then $0 < |x - a| < 10^6\delta$, so $|f(x) - L| < \epsilon$.

Actual $\Rightarrow$ 10^6**:** Fix $\epsilon > 0$. By **Actual**, there is an $\eta > 0$ such that, for all x, if $0 < |x - a| < \eta$, then $|f(x) - L| < \epsilon$. Let $\delta = 10^{-6}\eta$. Then $\delta > 0$ iff $\eta > 0$. For all x, if $0 < |x - a| < 10^6\delta = \eta$, $|f(x) - L| < \epsilon$.

From actual student papers:

Definition St1.

$$\lim_{x \to a} f(x) = L$$

means: For every $\epsilon > \delta$, the limit of $f(x)$ as x goes to a is L.

Definition St2.

$$\lim_{x \to a} f(x) = L$$

means: There exists a $\delta > 0$ and $\epsilon > 0$ such that $|x - a| < \delta$ and $|f(x) - L| < \epsilon$.

Definition St3.

$$\lim_{x \to a} f(x) = L$$

means:

$$\lim_{x \to a} f(x) = f(a).$$

Therefore $f(a) = L$.

Definition St4.

$$\lim_{x \to a} f(x) = L$$

means: For every $\epsilon > 0$, there is some $\delta > 0$ such that, for all x, if $0 < |x - a| < \delta$, then $|f(x) - L| < \epsilon$.

Definition St5.

$$\lim_{x \to a} f(x) = L$$

means: There exists some $\epsilon > 0$ *such that* $|x - a| = \delta$ $\delta > 0$ *as* $|\epsilon - \delta|$ *gets "small"* $f(x) = L$.

Definition St6.

$$\lim_{x \to a} f(x) = L$$

means: There exists a $f(\epsilon) \leq f(a) \leq f(\delta)$ *for some* ϵ, δ *in the domain of* $f(x)$. *I don't remember!!*

Definition S6.

$$\lim_{x \to a} f(x) = L$$

means: For every $\epsilon > 0$, *there is some* $\delta > 0$ *such that, for all* x, *if* $0 < |x - a| < \delta$, *then* $|f(x) - L| < \epsilon$.

Definition St7.

$$\lim_{x \to a} f(x) = L$$

means: For every $\epsilon > 0$, *there exist* $\delta > 0$ *such that if* $0 < |x - a| < \delta$, *then* $|f(x) - f(a)| < \epsilon$.

Definition St8.

$$\lim_{x \to a} f(x) = L$$

means: For every $\epsilon > 0$, *there is some* $\delta > 0$ *such that, for all* x, *if* $0 < |x - a| < 0$, *then* $|f(x) - L| < \epsilon$.

Definition St9.

$$\lim_{x \to a} f(x) = L$$

means: Given an $\epsilon > 0$, *but otherwise as small as we like, we can find that* $\delta > 0$ *such that* $0 < |x - a| < \delta$, *given* $|f(x) - L| < \epsilon$.

Definition St10.

$$\lim_{x \to a} f(x) = L$$

means: The limit of a function as it approaches a.

Definition St11.

$$\lim_{x \to a} f(x) = L$$

means: If $f(x)$ is within ϵ of L is within δ of a. So the closer we desire our $f(x)$ to be to L the smaller we must choose an δ, until it is clear that $f(x)$ approaches L as x approaches a.

Definition St12.

$$\lim_{x \to a} f(x) = L$$

means: For every $\epsilon > 0$, there is some $\delta > 0$ such that, for all x, if $0 < |x - a| < \delta$, then $|f(x) - L| < \epsilon$.

Definition St13.

$$\lim_{x \to a} f(x) = L$$

means: A continuous function has a limit at a if $|f(x) - f(a)| < \epsilon$, $\epsilon > 0$ when $|x - a| < \delta$ for some $\delta > 0$ and the limit $L = f(a)$.