MthT 430 Projects Chap 8e – More Adding sup and inf

More Understanding sup and inf

For $A \neq \emptyset$, it is useful to note the working characterizations of $\sup A$ and $\inf A$:

For $A \neq \emptyset$, $\sup A$ is the number α such that

$$\begin{cases} \text{For every } x \in A,\ x \leq \alpha, \\ \text{For every } \epsilon > 0, \text{ there is an } x \in A \text{ such that } x > \alpha - \epsilon. \end{cases}$$

For $A \neq \emptyset$, $\inf A$ is the number β such that

$$\begin{cases} \text{For every } x \in A,\ x \geq \beta, \\ \text{For every } \epsilon > 0, \text{ there is an } x \in A \text{ such that } x < \beta + \epsilon. \end{cases}$$

If f is a bounded function on $[0,1]$, we define

$$\sup f = \sup_{x \in [0,1]} f(x)$$

$$\inf f = \inf_{x \in [0,1]} f(x)$$

- Show that if f and g are bounded functions on $[0,1]$, then
$$\sup(f+g) \leq \sup f + \sup g.$$

- Give an example of bounded functions f and g on $[0,1]$ such that
$$\sup(f+g) < \sup f + \sup g.$$

- Show that if f and g are bounded functions on, then $[0,1]$,
$$\inf f + \inf g \leq \inf(f+g).$$

- Show that if f and g are bounded functions on, then $[0,1]$,
$$\inf f + \sup g \leq \sup(f+g).$$

The general result is that for two bounded bounded functions f and g on $[0,1]$

$$\begin{aligned} \inf f + \inf g &\leq \inf(f+g) \\ &\leq \inf f + \sup g \\ &\leq \sup(f+g) \\ &\leq \sup f + \sup g. \end{aligned}$$