

The Algebra Symposium: Discussion of Variables and Units

1. I went to Pompeii and bought the same number of salads and small pizzas. Salads cost two dollars each and pizzas cost six dollars each. I spent \$40 all together. Assuming that the equation $2S + 6P = 40$ is correct. Then

$$2S + 6P = 40.$$

Since $S = P$, I can write

$$2P + 6P = 40.$$

So

$$8P = 40.$$

The last equation says 8 pizzas is equal to \$40 so each pizza costs \$5.

What is wrong with the above reasoning? Be as detailed as possible. How would you try to help a student who made this mistake.

Discussion

The paradox is that the data told us that pizzas cost six dollars each but the calculation seems to show that each pizza costs \$5.

Let's examine the units of variables and constants in the equation

$$2S + 6P = 40.$$

S = number of salads,

P = number of pizzas,

$$2 = 2 \frac{\text{dollars}}{\text{salad}},$$

$$6 = 6 \frac{\text{dollars}}{\text{pizza}},$$

$$40 = 40 \text{ dollars.}$$

Thus the equation reads

$$2 \frac{\text{dollars}}{\text{salad}} S \text{ salads} + 6 \frac{\text{dollars}}{\text{pizza}} P \text{ pizzas} = 40 \text{ dollars.}$$

Using that $S = P$,

$$2P \text{ dollars} + 6P \text{ dollars} = 40 \text{ dollars,}$$

$$P = 5,$$

and $P = 5$, where the variable P represents the *number of pizzas*, not the *price per pizza*, in $\frac{\text{dollars}}{\text{pizza}}$.

Old Proportion Problems

Simple Proportion

2. If the interest upon a sum of money for 9 months is 318.69, what will be the interest for 11 $\frac{1}{2}$ months?
3. If 15 men can do a piece of work in 36 days, in how many days can they perform the same work with the assistance of 9 men more?

Units: 1 workpiece = (15 men) \cdot (36 days) = 15 \cdot 36 man–days. The question is

$$\begin{aligned} 1 \text{ workpiece} &= 15 \cdot 36 \text{ man–days} \\ &= (24 \text{ men}) \cdot (x \text{ days}). \end{aligned}$$

4. If a garrison of 200 men has provisions for 8 months, how many men must leave at the end of 5 months that the provisions remaining may last the rest 8 months longer?

Units: 1 commisary = (200 men) \cdot (8 provision–months).

The question is

$$\begin{aligned} 1 \text{ commisary} &= 200 \cdot 8 \text{ man–provision–months} \\ &= 200 \cdot 5 \text{ man–provision–months} + (200 - x) \cdot 8 \text{ man–provision–months}. \end{aligned}$$

Compound Proportions

5. If 11 men build 45 rods of wall in 6 days of 10 hours each, how many men will be required to build 81 rods of wall in 12 days of 11 hours each?

Units :

$$\begin{aligned} 45 \text{ rods} &= 11 \text{ men} \cdot 6 \text{ days} \cdot 10 \frac{\text{hours}}{\text{day}} \\ &= 11 \cdot 6 \cdot 10 \text{ man–hours}. \end{aligned}$$

The question is

$$\begin{aligned} 81 \text{ rods} &= x \text{ men} \cdot 12 \text{ days} \cdot 11 \frac{\text{hours}}{\text{day}} \\ &= x \cdot 12 \cdot 11 \text{ man–hours}. \end{aligned}$$

6. (oral exercise) If 7 men can dig 32 rods of ditch in 1 day, how many men will be required to dig 92 rods in $\frac{3}{4}$ day.