Stat 401 - 8/30/17

HW Sec 1.2 # 1, 10, 12, 16 due Wed 9/6

Sec 1.2 - Set Theory

def Null Set

- set with no elements.
- denote by ϕ or $\{ \}$

def Union

2 sets
- set of all elements that belong to at least one of the sets C_1 or C_2.
- denote by $C_1 \cup C_2$

ex $C_1 = \{1, 3, 4\}$ $C_2 = \{2, 4, 5\}$

$$C_1 \cup C_2 = \{1, 2, 3, 4, 5\}$$

Extension:

Let $C_1, C_2, \ldots, C_n, \ldots$ be sets.

Union = set of all elements that belong to at least one of $C_1, C_2, \ldots, C_n, \ldots$

$$C_1 \cup C_2 \cup \cdots \cup C_n \cup \cdots = \bigcup_{j=1}^{\infty} C_j$$

countable union.

ex) $C_1 = \{x : 0 \le x \le 1\}$

$C_2 = \{x : x = 2, 3, 4\}$

$C_1 \cup C_2 = \{x : 0 \le x \le 1, \text{or } 2, 3, 4\}$

ex) If $C_1 \subset C_2$ then $C_1 \cup C_2 = C_2$

ex) If $C_2 = \emptyset$ then $C_1 \cup C_2 = C_1$

ex) $C \cup C = C$

ex) $C_k = \{x : \frac{1}{k+1} \le x \le 1\}$ for

$k = 1, 2, 3, \ldots$

$\bigcup_{k=1}^{\infty} C_k = \{x : 0 < x \le 1\}.$ $\quad x \in [.1, \infty)$ $\quad 0 < x \le 1$

$C_1 = \{x : \frac{1}{2} \le x \le 1\}$

$C_{1000} = \{x : \frac{1}{1001} \le x \le 1\}$

def) Intersection.

- set of all elements that belong to both C_1 and C_2
- denoted by $C_1 \cap C_2$.

ex) $C_1 = \{1,2\}$ $C_2 = \{2,3\}$

$$C_1 \cap C_2 = \{2\}.$$

$$C_1 \cap C_2 \cap \cdots \cap C_n \cap \cdots = \bigcap_{j=1}^{\infty} C_j$$

countable intersection.

ex). $C_1 = \{(0,0), (0,1), (1,1)\}$

$C_2 = \{(1,1), (1,2), (2,1)\}$.

$C_1 \cup C_2 = \{(0,0), (0,1), (1,1), (1,2), (2,1)\}$

$= C_2 \cup C_1$

$C_1 \cap C_2 = \{(1,1)\}$

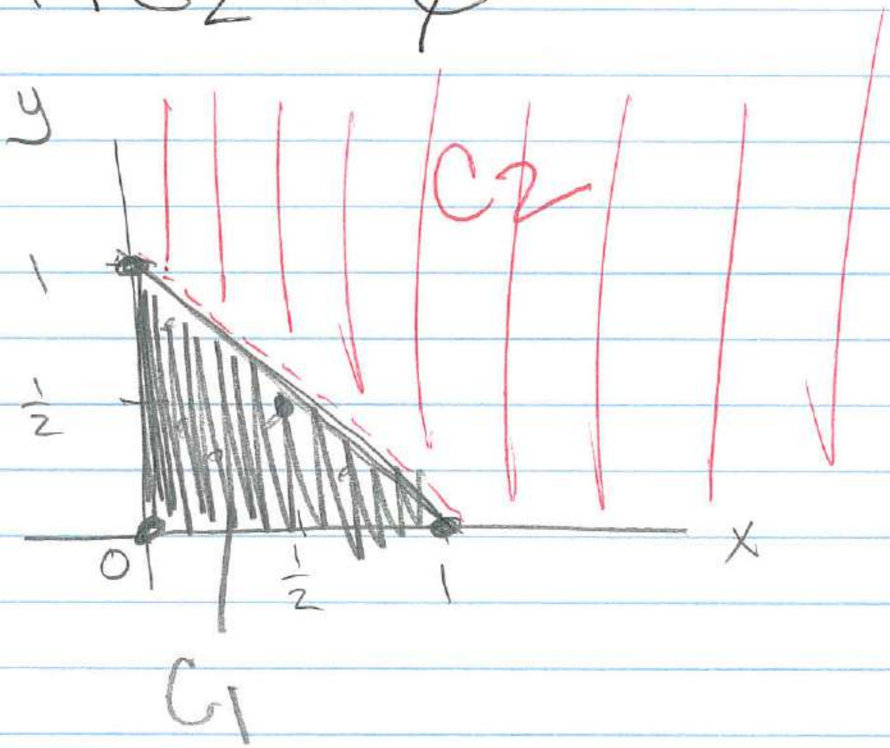
$= C_2 \cap C_1$

Suppose $x, y \in \mathbb{R}^+ \cup \{0\}$

ex). $C_1 = \{(x,y) : 0 \le x+y \le 1\}$.

$C_2 = \{(x,y) : 1 < x+y\}$.

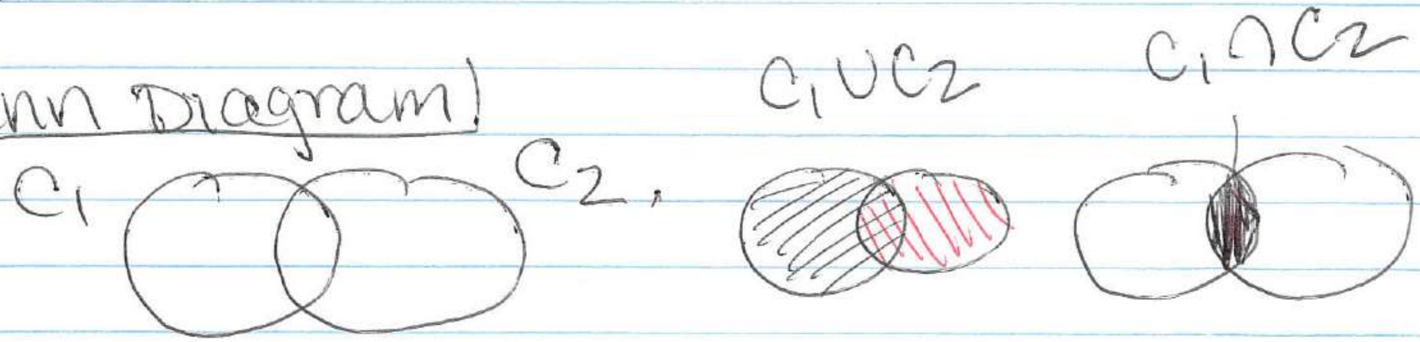
$C_1 \cap C_2 = \emptyset$



ex). If $C_k = \{x : 0 < x < \frac{1}{k}\}$, $k = 1, 2, 3, \ldots$

$$\bigcap_{k=1}^{\infty} C_k = \emptyset$$

Venn Diagram)



def] Space.

- totality of all elements is of interest.
- set of all elements under consideration = space.
- denote by $\mathscr{C}$, $\mathscr{D}$

script C   script D

ex]. toss coin 4 times
count # heads
Define space.

$$\mathscr{C} = \{0, 1, 2, 3, 4\}.$$

def]. Complement

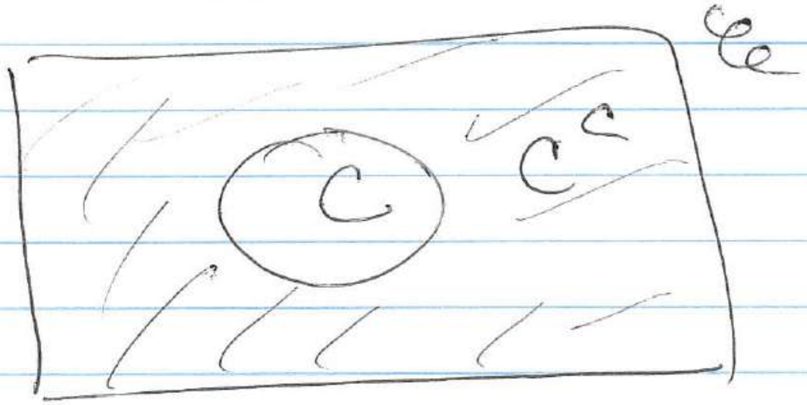
Let $\mathscr{C}$ be a space
Let $C \subset \mathscr{C}$
Complement is the set that consists of all elements in $\mathscr{C}$ that are not elements of C.

Denoted by C^c or C'

ex) Let $\mathscr{C} = \{0, 1, 2, 3\}$.

$$C = \{0, 1\}. \quad C^c = \{2, 3\}.$$

ex)



So
$$C \cup C^c = \mathscr{C}$$
$$C \cap C^c = \phi$$
$$C \cup \mathscr{C} = \mathscr{C}$$
$$C \cap \mathscr{C} = C$$

De Morgan's Laws

Let $\mathscr{C}$ be a space and $C_i \subset \mathscr{C}$, $i = 1, 2$

then $(C_1 \cap C_2)^c = C_1^c \cup C_2^c$. intersection law.

and $(C_1 \cup C_2)^c = C_1^c \cap C_2^2$ union law.

def). Set Functions.

- functions that map sets to real numbers.

ex) Let C be a set in 1-dim space.

Let $Q(C)$ be the # of points in C which correspond to positive integers.

If $C = \{x : 0 < x < 5\}$

then $Q(C) = 4$ 1, 2, 3, 4

If $C = \{-2, -1\}$. then $Q(C) = 0$

If $C = \{x : -\infty < x < 6\}$

then $Q(C) = 5$.

1, 2, 3, 4, 5