Stat 401 9/6/17

HW Sec 1.3 # 2, 4, 5, 12, 21 due Mon 9/11

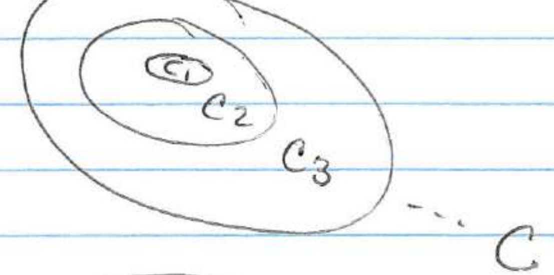
Properties of Probability Functions

(6a) If $\{C_n\}$ is a monotone increasing sequence then
$$\lim_{n\to\infty} \mathbb{P}(C_n) = \mathbb{P}\left(\bigcup_{n=1}^{\infty} C_n\right) = \mathbb{P}\left(\lim_{n\to\infty} C_n\right)$$

b) If $\{C_n\}$ is a monotone decreasing sequence then
$$\lim_{n\to\infty} \mathbb{P}(C_n) = \mathbb{P}\left(\bigcap_{n=1}^{\infty} C_n\right) = \mathbb{P}\left(\lim_{n\to\infty} C_n\right)$$

Proof of a) |

Suppose $C_n \uparrow C$



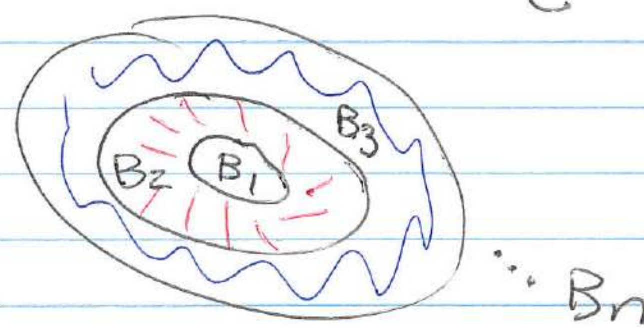
Let $B_1 = C_1$

$B_2 = C_2 \setminus C_1$

$B_3 = C_3 \setminus C_2$

$\vdots$

$B_n = C_n \setminus C_{n-1}$



So B_n is mutually exclusive and $\bigcup_{n=1}^{\infty} B_n = \bigcup_{n=1}^{\infty} C_n$

$$\mathbb{P}\left(\lim_{n\to\infty} C_n\right) \overset{\text{initial assumption}}{=} \mathbb{P}(C) \overset{\text{initial assumption}}{=} \mathbb{P}\left(\bigcup_{n=1}^{\infty} C_n\right)$$

$= \mathbb{P}\left(\bigcup_{n=1}^{\infty} B_n\right)$; how we defined B_n

$= \sum_{n=1}^{\infty} \mathbb{P}(B_n)$; since B_n is mutually exclusive.

$$= \lim_{N\to\infty} \sum_{n=1}^{N} \mathbb{P}(B_n)$$

$$= \lim_{N\to\infty} \left[\mathbb{P}(B_1) + \sum_{n=2}^{N} \mathbb{P}(B_n) \right]$$

$$= \lim_{N\to\infty} \left[\mathbb{P}(C_1) + \sum_{n=2}^{N} \left(\mathbb{P}(C_n) - \mathbb{P}(C_{n-1}) \right) \right]$$

$$= \lim_{N\to\infty} \Big[\cancel{\mathbb{P}(C_1)} + \underbrace{\cancel{\mathbb{P}(C_2)} - \cancel{\mathbb{P}(C_1)}}_{n=2} + \underbrace{\cancel{\mathbb{P}(C_3)} - \cancel{\mathbb{P}(C_2)}}_{n=3} + \cdots + \underbrace{\mathbb{P}(C_N) - \cancel{\mathbb{P}(C_{N-1})}}_{n=N} \Big]$$

$$= \lim_{N\to\infty} \mathbb{P}(C_N) \qquad \square$$

$$\begin{aligned} &\mathbb{P}(C_3) - \mathbb{P}(C_2) \\ + &\mathbb{P}(C_4) - \mathbb{P}(C_3) \\ + \cdots + &\mathbb{P}(C_{N-3}) - \mathbb{P}(C_{N-4}) \\ + &\mathbb{P}(C_{N-2}) - \mathbb{P}(C_{N-3}) \\ + &\mathbb{P}(C_{N-1}) - \mathbb{P}(C_{N-2}) \\ + &\mathbb{P}(C_N) - \mathbb{P}(C_{N-1}) \end{aligned}$$

7 $$\mathbb{P}\left(\bigcup_{n=1}^{\infty} C_n\right) \le \sum_{n=1}^{\infty} \mathbb{P}(C_n)$$

Subadditivity
Boole's Inequality.

Q How can we apply these rules?

Multiplication Rule

Suppose have k experiments.

Exper. 1 : n_1 outcomes
Exper. 2 : n_2 outcomes
Exper. 3 : n_3 outcomes
⋮
Exper. k : n_k outcomes.

Composite Experiment
(E1 followed by E2
followed by E3
... to Ek)
outcomes

$$= n_1 \cdot n_2 \cdot n_3 \cdot \ldots \cdot n_k$$

Consequences:

1. Suppose A has n elements
Interested in taking a k-tuple whose components are elements of A.

k-tuples:

$$\underbrace{n \cdot n \cdot \ldots \cdot n}_{k \text{ times.}} = n^k$$

ex) $A = \{1, 2, 3, 4\}$ $(n=4)$

2-tuple:

11	21	31	41
12	22	32	42
13	23	33	43
14	24	34	44

k

$16 = 4^2$

2. Suppose $k \leq n$.
How many k-tuples there are such that the components are distinct.

ex)
12 that are distinct.

$$\underset{\text{comp 1}}{\underline{n}} \quad \underset{\text{comp2}}{\underline{n-1}} \quad \underset{\text{comp3}}{\underline{n-2}} \quad \underset{\text{comp4}}{\underline{n-3}} \quad \ldots \quad \underset{\text{comp k}}{\underline{n-(k-1)}}$$

def <u>Permutation</u>

$$P^n_k = n(n-1)(n-2)\cdots(n-(k-1))$$

$$= \frac{n!}{(n-k)!}$$ order matters.

(3) Suppose ~~not~~ order not important and no repeats. Find # of subsets of k elements there are taken from A.

ex) $A = \{1, 2, 3, 4\}$.
Arrange in groups of 2.

$n = 4 \qquad k = 2$

(1) 1 2 = 2 1
(2) 1 3 = 3 1
(3) 1 4 = 4 1
(4) 2 3 = 3 2
(5) 2 4 = 4 2
(6) 3 4 = 4 3

6 options

<u>def</u>)

Combination

$$= C^n_k = \binom{n}{k} = \frac{n!}{(n-k)!\,k!} = \frac{P^n_k}{k!}$$

n choose k

<u>Remark</u>)

Binomial Coefficient.

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$

Cards - 4 suits
- heart - red
- diamond - red
- spade - black
- club, - black.

each with 13 cards.

Ace 2, 3, 4, 5, 6, 7, 8, 9, 10,

J, Q, K

face cards.

total of 52 cards.

Ex 1) $\mathscr{C}_e$ = { all the outcomes of drawing a card from a deck of 52 cards }.

Assign each outcome a probability of $\frac{1}{52}$

a) E_1 = { spades } ← 13 elements

$$\mathbb{P}(E_1) = \frac{13}{52}$$

b) E_2 = { kings } ← 4 elements.

$$\mathbb{P}(E_2) = \frac{4}{52} = \frac{1}{13}$$

Ex 2) $\mathscr{C}_e$ = { five cards }.

Interested in a flush.

$\binom{52}{5}$ # ways to select 5 cards.

ways get a flush. $\underset{\text{suit}}{\binom{4}{1}}\underset{\text{cards}}{\binom{13}{5}}$

$$\mathbb{P}(\text{flush}) = \frac{\binom{4}{1}\binom{13}{5}}{\binom{52}{5}}$$

$$= \frac{\#\text{ ways to get what want}}{\text{total ways}}$$

$$\approx 0.00198$$