Stat 401 - 9/8/17
Sec 1.3 HW due Monday.

Sec 1.4 - Conditional Probability and Independence

Setup: $\mathcal{C}$ = sample space.
$C_1, C_2, \dots, C_n$ = events in $\mathcal{C}$

want to restrict ourselves to event C_1.

want to know prob. of other events occurring given our restricted sample space.

ex) Suppose I want to paint a room 2 different colors.

Possible colors: purple
black
red
yellow
blue
pink.

Suppose purple = color #1.

Now the chance for color #2 may be different.

def| Conditional Probability (2 Events C_1 and C_2).

- The probability C_2 occurs when it is known that event C_1 has already occurred.

Denoted by

$$\mathbb{P}(C_2 \mid C_1) \triangleq \frac{\mathbb{P}(C_1 \cap C_2)}{\mathbb{P}(C_1)}$$

what we are interested in

given

prior info that. Already occurred

defined as

where $\mathbb{P}(C_1) > 0$

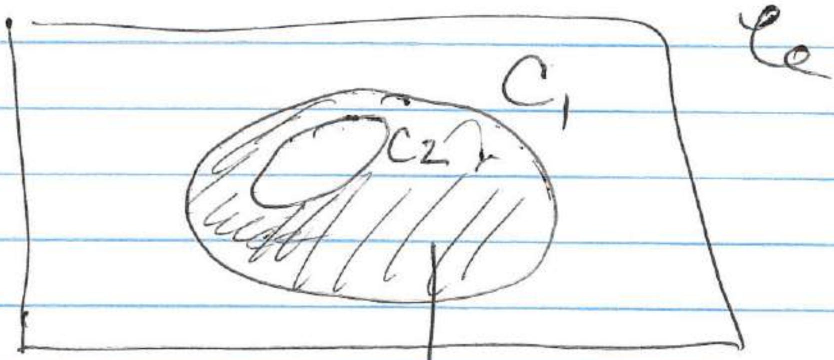
Properties

1. $\mathbb{P}(C_2 \mid C_1) \geq 0$
2. $\mathbb{P}\left(\bigcup_{j=2}^{\infty} C_j \,\middle|\, C_1\right) = \sum_{j=2}^{\infty} \mathbb{P}(C_j \mid C_1)$

where $C_2, C_3, C_4, \ldots$ are mutually exclusive events.

(3) $\mathbb{P}(C_1 | C_1) = 1$

(4) $\mathbb{P}(C_2^c | C_1) = 1 - \mathbb{P}(C_2 | C_1)$



$C_2^c | C_1$

Ex 1 $\mathbb{P}(C_2 | C_1)$

$C_1 = \geq 4$ spades

$C_2 = 5$ spades (hand of 5 cards).

$$\mathbb{P}(C_2 | C_1) = \frac{\mathbb{P}(C_1 \cap C_2)}{\mathbb{P}(C_1)}$$

$$= \frac{\mathbb{P}(C_2)}{\mathbb{P}(C_1)}$$

$C_1 \cap C_2$
$\geq 4 \quad 5$
$= 5$ spades
$= C_2$.

$$\mathbb{P}(C_2) = \frac{\binom{13}{5}}{\binom{52}{5}}$$

← # ways of what I want

← total # ways.

$$\mathbb{P}(C_1) = \frac{\binom{13}{4}\binom{39}{1} + \binom{13}{5}}{\binom{52}{5}}$$

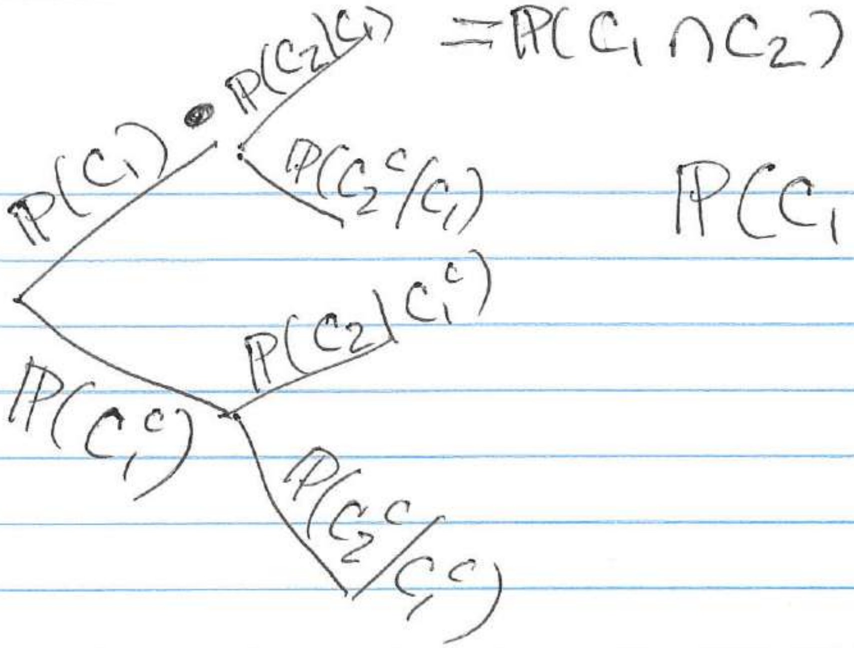
$$\frac{\mathbb{P}(C_2)}{\mathbb{P}(C_1)} = \frac{\binom{13}{5}}{\binom{13}{4}\binom{39}{1} + \binom{13}{5}}$$

$$\approx 0.0441$$

$$= \frac{\frac{13!}{5!(8)!}}{\frac{13!}{4!\,9!} \cdot \frac{39!}{1!\,38!} + \frac{13!}{5!\,8!}}$$

<u>def</u> Multiplication Rule

$$\mathbb{P}(C_1 \cap C_2) = \mathbb{P}(C_1) \cdot \mathbb{P}(C_2 | C_1)$$



$$\mathbb{P}(C_1 \cap C_2^c) = \mathbb{P}(C_1) \cdot \mathbb{P}(C_2^c|C_1)$$

Ex 2 8 chips 3red 5blue

$C_1 =$ red $\quad C_2 =$ blue

Find $\mathbb{P}(C_1 \cap C_2)$.

$$\mathbb{P}(C_1) = \frac{3}{8} = \frac{\binom{3}{1}}{\binom{8}{1}}$$

$$\mathbb{P}(C_2 | C_1) = \frac{5}{7}$$

$$\mathbb{P}(C_1 \cap C_2) = \mathbb{P}(C_1) \cdot \mathbb{P}(C_2 | C_1)$$

$$= \frac{3}{8} \cdot \frac{5}{7} = \boxed{\frac{15}{56}}$$

Ex 3) 4^{th} spade on 7^{th} draw

$$\underbrace{\_\ \_\ \_\ \_\ \_\ \_}_{\text{3 are spades}}\ \_$$

3 are spades
locations unknown.

spade #4

Define C_1 = 3 spades in first 6 draws

C_2 = spade on 7^{th} draw.

Find $\mathbb{P}(C_1 \cap C_2)$

$$\mathbb{P}(C_1) = \frac{\binom{13}{3}\binom{39}{3}}{\binom{52}{6}} \approx 0.1284$$

$$\mathbb{P}(C_2 \mid C_1) = \frac{10}{46} \approx 0.2174$$

$$\mathbb{P}(C_1 \cap C_2) = (0.1284)(0.2174)$$

$$\approx 0.0279$$

<u>def]</u> General Multiplication Rule (n events).

$$\mathbb{P}(C_1 \cap C_2 \cap \cdots \cap C_n)$$
$$= \mathbb{P}(C_1) \cdot \mathbb{P}(C_2 | C_1) \cdot \mathbb{P}(C_3 | C_1 \cap C_2) \cdot \ldots \cdot \mathbb{P}(C_n | C_1 \cap C_2 \cap \cdots \cap C_{n-1})$$