Stat 401 - 9/29/17

Exam 1 on Friday Oct 6

Sec 2.1 - Distributions of 2 variables

examples:

1. McDonalds
   - → Type of Burger
   - → Size Fries.
2. Hardness and Tensile strength of cold-drawn copper.
3. Toss a coin 3 times.

Define a joint RVs (X_1, X_2) to be

$$(X_1, X_2) = \left(\text{\# heads on } 1^{st} \text{ 2 tosses}, \ \text{\# heads on } 1^{st} \text{ 3 tosses} \right)$$

Space consists of ordered number pairs.

$$\mathcal{D} = \{(0,0), (0,1), (1,1), (1,2), (2,2), (2,3)\}$$

$$\mathcal{C} = \{TTT, TTH, THT, HTT, THH, HTH, HHT, HHH\}$$

$$\mathcal{C} \xrightarrow{X} \mathcal{D}$$

def) Random Vector.

Given a random experiment with sample space $\mathcal{C}$, consider 2 RVs X_1, X_2. We say

$\vec{X} = (X_1, X_2)$ is a random vector.

the space of (X_1, X_2) is the set of ordered pairs

$$\mathcal{D} = \{ (x_1, x_2) : x_1 = X_1(c),\ x_2 = X_2(c),\ c \in \mathcal{C} \}$$

Similar to Sec 1.5, $\vec{X}$ induce a probability set function on $\mathcal{D}$ by

$$\underbrace{P_{X_1,X_2}(D) = \mathbb{P}[(X_1, X_2) \in D] \text{ where } D \in \mathcal{D}}_{\text{distribution of } \vec{X}}$$

(Joint) CDF of $\vec{X}$:

$$F_{X_1,X_2}(x_1,x_2) = \mathbb{P}\left[\{X_1 \le x_1\} \cap \{X_2 \le x_2\}\right]$$

$$= F_{1,2}(x_1,x_2) \qquad = \mathbb{P}\left[X_1 \le x_1,\ X_2 \le x_2\right]$$

for all $(x_1, x_2) \in \mathbb{R}^2$

Discrete Random Vectors

→ D is finite ~~&~~ or countable.

def] (joint) probability mass function (pmf) as

$$p_{X_1,X_2}(x_1,x_2) = \mathbb{P}\left[X_1 = x_1,\ X_2 = x_2\right]$$

$$p_{\cancel{X_2},X_1}(x_1,x_2) = \mathbb{P}\left[X_2 = x_1,\ X_1 = x_2\right]$$

for all $(x_1, x_2) \in D$

Properties:

1. $0 \le p_{X_1,X_2}(x_1,x_2) \le 1$
2. $\sum_D \sum p_{X_1,X_2}(x_1,x_2) = 1$
3. $\mathbb{P}[(X_1,X_2) \in \underbrace{A}_{\text{event}}] = \sum_A \sum p_{X_1,X_2}(x_1,x_2)$ where $A \subseteq D$

ex) Coin Toss

8 equally likely outcomes in $\mathcal{C}$.

D_{X_1} \ D_{X_2}	0	1	2	3	marg Dist X_1
0	$\frac{1}{8}$	$\frac{1}{8}$	0	0	$\frac{2}{8}$
1	0	$\frac{2}{8}$	$\frac{2}{8}$	0	$\frac{4}{8}$
2	0	0	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{2}{8}$
marg Dist X_2	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	

Space for X_1 : $D_{X_1} = \{0, 1, 2\}$

" " X_2 : $D_{X_2} = \{0, 1, 2, 3\}$

$\to \mathbb{P}[X_1 = 1, X_2 = 1]$

All prob. add up to 1

marg Dist X_2

Find $\mathbb{P}[X_1 \le 1, X_2 \le 2]$

$$= F_{1,2}(1,2)$$

$$= \sum_{x_1 \le 1} \sum_{x_2 \le 2} p_{1,2}(x_1, x_2)$$

$$= p_{1,2}(0,0) + p_{1,2}(0,1) + p_{1,2}(0,2) + p_{1,2}(1,0) + p_{1,2}(1,1) + p_{1,2}(1,2)$$

$$= \frac{1}{8} + \frac{1}{8} + 0 + 0 + \frac{2}{8} + \frac{2}{8}$$

$$= \frac{6}{8} = \frac{3}{4}$$

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we can consider each RV individually.

CDF of X_1:

$$F_{X_1}(x_1) = \mathbb{P}[X_1 \le x_1]$$

$$= \mathbb{P}[X_1 \le x_1, -\infty < X_2 < \infty]$$

$$= \lim_{x_2 \to \infty} \mathbb{P}[X_1 \le x_1, -\infty < X_2 \le x_2]$$

$$= \lim_{x_2 \to +\infty} F_{X_1, X_2}(x_1, x_2)$$

ex)

$$F_{X_1}(x_1) = \sum_{k=-\infty}^{x_1} \sum_{x_2=-\infty}^{\infty} p_{X_1,X_2}(k, x_2)$$

PMF for X_1 ← marginal Dist of X_1

$$p_{X_1}(x_1) = \mathbb{P}[X_1 = x_1]$$

$$= \mathbb{P}[X_1 = x_1,\ X_2 \in D_{X_2}]$$

$$= \sum_{x_2 \in D_{X_2}} \mathbb{P}[X_1 = x_1,\ X_2 = x_2]$$

$$= \sum_{x_2 \in D_{X_2}} p_{X_1,X_2}(x_1, x_2)$$

PMF for X_2 ← marginal Dist of X_2

$$p_{X_2}(x_2) = \sum_{x_1 \in D_{X_1}} p_{1,2}(x_1, x_2)$$

marg dist X_1

x_1	0	1	2
$p_1(x_1)$	$\frac{2}{8}$	$\frac{4}{8}$	$\frac{2}{8}$

marg dist X_2

x_2	0	1	2	3
$p_2(x_2)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$