Stat 401 - 11/8/17 Review Day

Structure

Undergrad: 4 Quest. + 1 Bonus

3 from Ch 2 ... Ch 2

1 from Ch 3 (Sec 3.1; 3.2)

Grad: 5 Quest
- 4 from Ch 2
- 1 from Ch 3 (Sec 3.1, 3.2)

Topics
- How to find CDF / PDF
- Transformations
- Covariance and what it means (help to describe the linear relationship between 2 RVs).

$$Cov(X,Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$$

- Independence (def, meaning)
- Find marginal dist.
- Find conditional dist.
- Find MGF $M_X(t) = \mathbb{E}[e^{tX}]$

- Linear Combinations of RVs.

$$\mathbb{E}\left[\sum a_i X_i\right] = \sum a_i \mathbb{E}[X_i]$$

ex) $\mathbb{E}[X_1 + 2X_2 - 4X_3]$

$= \mathbb{E}[X_1] + 2\mathbb{E}[X_2] - 4\mathbb{E}[X_3]$

$$\mathrm{Var}\left[\sum a_i X_i\right] = \sum a_i^2 \mathrm{Var}[X_i]$$

if X_i's are indep.

ex) $\mathrm{Var}[3X_1 - 2X_2]$

$= \mathrm{Var}[3X_1 + (-2)X_2]$

$\overset{\text{indep}}{=} 3^2 \mathrm{Var}[X_1] + (-2)^2 \mathrm{Var}[X_2]$

$= 9\mathrm{Var}[X_1] + 4\mathrm{Var}[X_2]$

Always

If not indep

$$\mathrm{Var}[aX + bY] = a^2\mathrm{Var}[X] + b^2\mathrm{Var}[Y] + 2ab\,\mathrm{Cov}(X,Y)$$

$$\mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n} X_i\right]$$

$$\mathrm{Var}\left[\frac{1}{n}\sum X_i\right] \text{ if } X_i\text{'s indep.}$$

- Definition of iid
  - indep. and identically distributed.
- Unbiased Estimators.

  If T is an estimator of a parameter θ then if

  $$\mathbb{E}[T] = \theta \text{ it is unbiased.}$$

  <u>ex)</u> $\bar{X}$ is unbiased for μ since

  $$\mathbb{E}[\bar{X}] = \mu$$

- Distributions
  - Bernoulli / Binomial
  - Geometric / Neg. Binomial
  - Poisson

DO NOT MEMORIZE

- pmfs / pdfs of dist. (but know parameters + what stand for)
- MGF
- $\mathbb{E}[X]$ $Var[X]$ for dist.

Study Guide

Sec 2.1 #1

$$p(x_1, x_2) = \frac{x_1 + x_2}{30} \qquad \begin{array}{l} x_1 = 0,1,2,3 \\ x_2 = 0,1,2. \end{array}$$

c) Find $F_1(x_1)$

$$F(x_1) = \mathbb{P}[X_1 \le x_1]$$

$$= \mathbb{P}[X_1 \le x_1, -\infty < X_2 < \infty]$$

$$= \sum_{k=0}^{x_1} \sum_{x_2=0}^{2} \frac{k + x_2}{30}$$

$$= \cdots$$

$$= \begin{cases} 0 & , \quad x_1 < 0 \\ \dfrac{(x_1+1)(x_1+2)}{20} & , 0 \le x_1 < 3 \\ 1 & , \quad 3 \le x_1 \end{cases}$$

if $x_1 = 0$

$\frac{1 \cdot 2}{20} = \frac{1}{10}$

if $x_1 = 2 = \frac{3 \cdot 4}{20} = \frac{12}{20} = \frac{6}{10}$

$$\sum_{k=0}^{n} (k+1) = \frac{(k+1)(k+2)}{2}$$

$$\sum_{k=0}^{n} k = \frac{k(k+1)}{2}$$

d)

x_1	0	1	2	3
$p(x_1)$	$\frac{1}{10}$	$\frac{1}{5} = \frac{2}{10}$	$\frac{3}{10}$	$\frac{2}{5} = \frac{4}{10}$

$$F(x_1) = \begin{cases} 0, & x_1 < 0 \\ \frac{1}{10}, & 0 \le x_1 < 1 \\ \frac{3}{10}, & 1 \le x_1 < 2 \\ \frac{6}{10}, & 2 \le x_1 < 3 \\ 1, & 3 \le x_1 \end{cases}$$