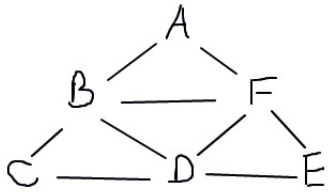
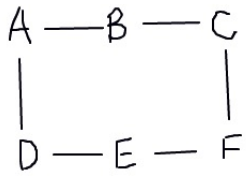
Problem 1. A cycle is a path that starts and ends in the same place. We call a cycle "Eulerian" if it visits every edge exactly once. Can you find an Eulerian cycle for the following graphs? (Why or why not?)

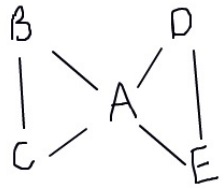
(a)



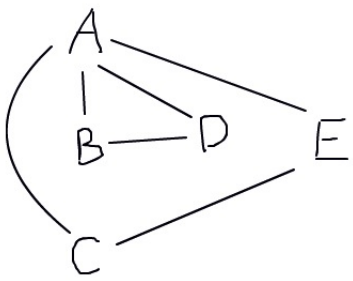
(b)



(c)

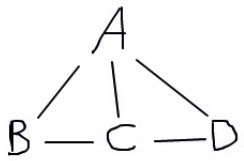


(d)

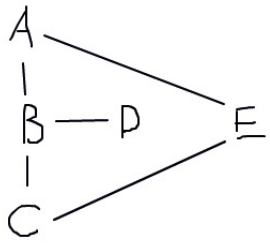


Problem 2. Can you find an Eulerian cycle in these graphs? Can you guess why or why not? Explain your answer to someone in your group. Together, write a short explanation.

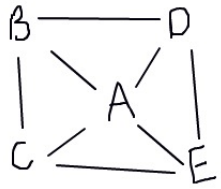
(a)



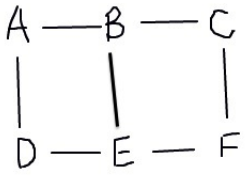
(b)



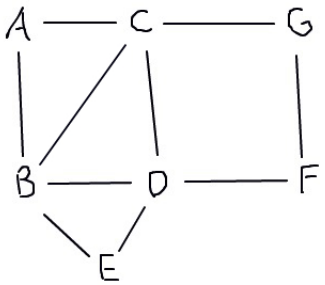
(c)



(d)



Problem 3. Look at the following graph. Draw the cycle that goes $A \to B \to C \to D \to B \to E \to D \to F \to G \to C \to A$. Is it an Eulerian cycle? For each vertex, count the number of times this path "enters" and "exits" the vertex. What do you notice?



enters A:
exits A:

enters B:
exits B:

enters C:
exits C:

enters D:
exits D:

enters E:
exits E:

enters F:
exits F:

enters G:
exits G:

Problem 4. If a vertex has odd degree, why can't every one of its edges belong to a cycle? Explain your answer to someone in your group. Together, write a short explanation. [Hint: Use the previous problem.]

Problem 5. Using the last problem, explain why a graph with an Eulerian cycle must have only vertices of even degree.

Problem 6. Lets say we are having a party with 10 people, and we want to know how many friends each person will have at the party. Let's figure out why two people at the party have the same number of friends.

(a) Can you turn this problem into one about graphs? What are the vertices? What about the edges?

(b) What happens if one person has no friends at the party? What about if one person has 9 friends? Can both of these happen at once?

(c) Can you figure out why two people at the party have the same number of friends at the party? Explain your answer to someone in your group. Together, write a short explanation.

Problem 7. Can you write the digits 1-9 in a row so that the sum of adjacent values are divisible by 5, 7, or 13? For example, $4, 9, 1$ works since $4+9 = 13$ is divisible by 13 and $9+1 = 10$ is divisible by 5 but $1, 6, 2$ doesn't work since $6+2 = 8$ isn't divisible by $5, 7$, or 13. [Hint: Turn the problem into a graph! What show the vertices be? What about the edges?]