Problem 1. Let's say there are 15 people in a class. Is it possible that none of them have their birthday in the same month? Explain your answer to someone in your group. Together, write a short explanation.

Problem 2. Let's pretend there are 27 people in a class. Is it possible that no month has more than two student birthdays in that month? Explain your answer to someone in your group. Together, write a short explanation.

Problem 3. Can you explain why two people in our classroom right now were born on the same day of the week? Explain your answer to someone in your group. Together, write a short explanation.

Problem 4. A standard game of pool is played with 15 balls (and one cue ball), on a table with 6 pockets. Why will there always be at least 3 balls in one of the pockets at the end of the game? Explain your answer to someone in your group. Together, write a short explanation.

Problem 5. Everyday before school, I pick out two socks. Unfortunately, since my closet is poorly lit and school starts early in the morning, it is always too dark to see what color of sock I am drawing. If there are 2 different colors of socks that I can draw, how many socks must I draw before I'm sure to draw 2 socks of the same color? Explain your answer to someone in your group. Together, write a short explanation.

Problem 6. In the winter, I also like to pick out mittens before school. Since everyone can see my mittens, I have not just 2, but 4 colors of mittens to choose from. If I can't see what color I am picking, how many mittens will I have to pull out before I'm sure I have a matching pair? Explain your answer to someone in your group. Together, write a short explanation.

Problem 7. When I'm late for school, I have to try to find socks and mittens as fast as possible, so I grab a sock from my sock draw with one hand while at the same time grabbing a mitten from my other drawer with the other hand.

(a) How many times will I have to grab socks and mittens before I know for sure that I have a matching pair of each? Explain your answer to someone in your group. Together, write a short explanation.

(b) My socks can be white or black, and my mittens can be white, black, tan, or blue. How many times will I have to pull a sock and a mitten if I want to have my mittens and socks match eachother? Explain your answer to someone in your group. Together, write a short explanation. (Optional: draw a picture or graph to explain)

Problem 8. Let's say I want to organize a math competition. I want to have 5 teams, and I want each team to play three contests against another team. Why is this a problem? Explain your answer to someone in your group. Together, write a short explanation.

Problem 9. Ben is having a party, and invites over some of his friends: Janet, Edgar, Matt, and Shikhar. At the party, people shake hands many times. Janet only shakes hands with Edgar. She wants to know how many total handshakes occurred so she goes around asking who else shook hands. As it turns out, Edgar shook hands with everyone, and Shikhar only shook hands with two people. Matt only shook hands with Edgar.

(a) Can you turn this problem into a graph? (Hint: Use the people as the vertices)

(b) How many handshakes occurred that evening?

(c) Janet goes around and asks each person how many hands they shook, and then sums up those numbers. How many handshakes does she count this way?

(d) The "degree" of a vertex is the number of edges coming out of it. Can you explain why the sum of the degrees of the vertices will be twice the number of edges?

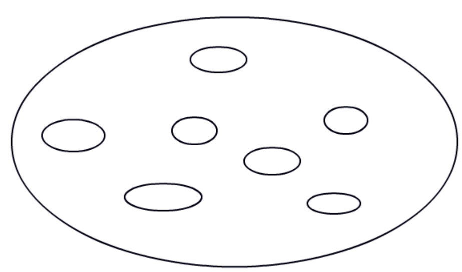
(e) Can you explain why the sum of degrees of all the vertices is always even?

***Problem 10.** Suppose that we have a party with 33 people and people shake hands with eachother. Why is it that two people have to shake the same number of hands? Explain your answer to someone in your group. Together, write a short explanation.

***Problem 11.** Let's say there are six people at a party. Two people are "strangers" if they haven't met before, and "friends" if they have. Can you explain why it is always the case that there will either be 3 people who are all mutual strangers, or 3 people who are all mutual friends (or both)? [Hint: Try to set up a graph to see if this is true: For each person, draw a vertex. Using colored pens or board markers, choose two different colors–one to mean "friends" and one to mean "strangers." Now connect each person with each other person with one of the colors. Are there always three people connected by the same color? Can you explain why?)

Extra: **Problem 12.** Let's say there is a lake that has 7 islands in it. Every island has 1,3 or 5 bridges, and the bridges connect either to the shore or to other islands.

(a) Can you put bridges in the following picture so it satisfies the above condition?



(b) Is there a way of placing bridges so they satisfy the above conditions and no bridge connects to the shore? Explain your answer to someone in your group. Together, write a short explanation.