MthT 435 **Problem Set One** **January 19, 2010**

For the first four problems $\mathcal{S}$ is the set of positive integers which, when divided by 4 have remainder 1,

$$\mathcal{S} = \{n : n = 4q + 1, q \geq 0\}.$$

1. Show that the product of two elements of $\mathcal{S}$ is also in $\mathcal{S}$. We say that $\mathcal{S}$ is closed under multiplication.

2. Call p an $\mathcal{S}$-prime if p is in $\mathcal{S}$ and for any d in $\mathcal{S}$, if $d|p$ then $d = 1$ or $d = p$. List all the numbers in $\mathcal{S}$ less than 101. Use the Sieve of Eratosthenes to circle the $\mathcal{S}$-primes and lightly cross through the $\mathcal{S}$-composites in this range. [Only five of these numbers are $\mathcal{S}$-composite.]

3. Adapt the proof that ordinary integers factor into ordinary primes to prove that any number in $\mathcal{S}$ is a product of $\mathcal{S}$-primes. [Regard 1 as the empty product.]

4. If p is an odd prime number in the ordinary integers show that:

   (a) if p lies in $\mathcal{S}$, then p is an $\mathcal{S}$-prime.

   (b) if p does not lie in $\mathcal{S}$, then p^2 does lie in $\mathcal{S}$ and p^2 is an $\mathcal{S}$-prime.

5. Using two numbers p and q as in problem 4. (b), construct a number in $\mathcal{S}$ with two factorizations into $\mathcal{S}$-primes which are not the same up to order. This means the uniqueness part of the Fundamental Theorem of Arithmetic is false for $\mathcal{S}$.

   Is it true in $\mathcal{S}$ that if an $\mathcal{S}$-prime divides a product it divides one of the factors?

6. Use the Euclidean algorithm to calculate the gcd of 48 and 30 and the gcd of 19,800 and 8,316.

7. Write $\gcd(48, 30)$ in the form $48x + 30y$.