MthT 435 **Problem Set Two** **February 9, 2010**

8. Let R be the ring of 2×2-matrices with real entries. Let

$$A = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 0 \\ b & 1 \end{pmatrix}, \qquad C = \begin{pmatrix} 1 & -a \\ 0 & 1 \end{pmatrix}.$$

   (a) Show C is a two-sided inverse of A.

   (b) Find a matrix X in R such that $AX + B = 0$ and show that any solution to this equation is equal to this X.

   (c) Find a matrix Z in R such that $ZA + B = 0$ and show that any solution to this equation is equal to this Z.

   Note that $X \neq Z$.

9. Let $Z/6$ be the ring of integers modulo 6. We denote the elements of $Z/6$ by $\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}$. Let $S \subset Z/6$ be the set $S = \{\bar{0}, \bar{2}, \bar{4}\}$.

   (a) Write the addition and multiplication table for the set S using the operations $+$ and $\cdot$ defined in $Z/6$. Do your tables confirm that S is closed under $+$ and $\cdot$?

   (b) The multiplicative identity, $\bar{1} \in Z/6$ is not an element of S, so S is not a subring of $Z/6$. Still we can ask, does some element of S satisfy the definition of a multiplicative identity? If so, which element? Is S a ring?

10. Show that, if n is a composite integer, then the ring Z/n has zero divisors. That is, show there are nonzero elements $a, b \in Z/n$ with $ab = 0$.

11. Show that, if p is a prime integer, then Z/p is a field. That is, show that every nonzero element of Z/p has a multiplicative inverse. [Recall that, for an integer a with $0 \leq a < p$, $\gcd(a, p) = 1$. Hence there are integers x, y with $ax + by = 1$.]