12. Prove from the axioms for a vector space $V$ over a field $F$ that if $a \in F$ and $v \in V$, then $(-a)v = -(av)$.

13. If $V$ is a vector space over a field $F$ and $U$ is a subset of $V$ which is closed under addition and scalar multiplication, i.e.

$$v, w \in U, \ a \in F \implies v + w \in U, \ av \in U,$$

then show that

$$v \in U \implies -v \in U.$$

14. Let $V$ be a vector space over the field of real numbers. Use the axioms to show that a vector $v \in V$ such that $v = -v$ must be the zero vector, $v = \vec{0}$. [The fact that $\mathbb{R}$ and $\mathbb{Q}$ have the element $\frac{1}{2}$ and $\frac{1}{2} \cdot 2 = 1$ is useful.]

15. Let $V = \mathbb{R}^3$ and let $U = \{(x_1, x_2, x_3) : x_1 + x_2 + x_3 = 0\}$. Show that $U$ is a subspace. It is enough to show:

(a) if $v \in U$ and $w \in U$, then $v + w \in U$;

(b) if $v \in U$ and $a \in \mathbb{R}$, then $av \in U$.

16. Find a basis for the vector space $U$ of problem 15.

17. If $V$ is a vector space over $\mathbb{Z}/2$ of dimension 3, how many vectors are in $V$?

18. The ring $\mathbb{R}[X]$ of polynomials with real coefficients can be regarded as a vector space $V$ over $\mathbb{R}$ by forgetting about polynomial multiplication. Let $U = \{p \in V : \deg p \leq 3\}$. Give a basis for $U$ over $\mathbb{R}$. 