MthT 435 **Problem Set Three** **March 2, 2010**

12. Prove from the axioms for a vector space V over a field F that if $a \in F$ and $v \in V$, then $(-a)v = -(av)$.

13. If V is a vector space over a field F and U is a subset of V which is closed under addition and scalar multiplication, *i.e.*

$$v, w \in U,\ a \in F \implies v + w \in U,\ av \in U,$$

then show that

$$v \in U \implies -v \in U.$$

14. Let V be a vector space over the field of real numbers. Use the axioms to show that a vector $v \in V$ such that $v = -v$ must be the zero vector, $v = \vec{0}$. [The fact that $\mathbb{R}$ and $\mathbb{Q}$ have the element $\frac{1}{2}$ and $\frac{1}{2}2 = 1$ is useful.]

15. Let $V = \mathbb{R}^3$ and let $U = \{(x_1, x_2, x_3) : x_1 + x_2 + x_3 = 0\}$. Show that U is a subspace. It is enough to show:

    (a) if $v \in U$ and $w \in U$, then $v + w \in U$,

    (b) if $v \in U$ and $a \in R$, then $av \in U$.

16. Find a basis for the vector space U of problem 15.

17. If V is a vector space over $\mathbb{Z}/2$ of dimension 3, how many vectors are in V?

18. The ring $\mathbb{R}[X]$ of polynomials with real coefficients can be regarded as a vector space V over $\mathbb{R}$ by forgetting about polynomial multiplication. Let $U = \{p \in V : \deg p \leq 3\}$. Give a basis for U over $\mathbb{R}$.