

19. Find a polynomial with integer coefficients which has a root equal to

$$1 + \sqrt{7},$$

$$\sqrt[3]{2} + \sqrt{3}.$$

20. Show that  $-3$  is a triple root of the equation

$$X^4 + 8X^3 + 18X^2 - 27 = 0.$$

What is the other root?

21. Show that  $1$  and  $1/2$  are double roots of the equation

$$4X^5 - 23X^3 + 33X^2 - 17X + 3 = 0.$$

What is the other root?

22. Show that  $X^3 - 3X - 1 = 0$  has no rational root. Could this polynomial be reducible in  $\mathbb{Q}[X]$ ?

Each of the following equations has one or more rational roots. In each case find all the roots.

23.  $X^3 - X^2 - 14X + 24 = 0.$

24.  $2X^4 + 7X^3 - 2X^2 - X = 0.$

25. Find a quadratic polynomial  $f(X) = aX^2 + bX + c \in \mathbb{Z}[X]$  with  $f(-\frac{1}{2} + \frac{1}{2}\sqrt{-3}) = 0$  and for which  $a, b,$  and  $c$  have no common factor greater than 1..