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DERIVATIVES

function	derivative
x^n	nx^{n-1}
e^x	e^x
$a^x (a > 0)$	$a^x \ln a$
$\ln x$	$\frac{1}{x}$
$\log_a x$	$\frac{1}{x \ln a}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\operatorname{cosech} x$	$-\operatorname{cosech} x \coth x$
$\operatorname{sech} x$	$-\operatorname{sech} x \tanh x$
$\coth x$	$-\operatorname{cosech}^2 x$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cosh^{-1} x (x > 1)$	$\frac{1}{\sqrt{x^2-1}}$
$\tanh^{-1} x (x < 1)$	$\frac{1}{1-x^2}$
$\coth^{-1} x (x > 1)$	$-\frac{1}{x^2-1}$

Product Rule

$$\frac{d}{dx}(u(x) v(x)) = u(x) \frac{dv}{dx} + v(x) \frac{du}{dx}$$

Quotient Rule

$$\frac{d}{dx} \left(\frac{u(x)}{v(x)} \right) = \frac{v(x) \frac{du}{dx} - u(x) \frac{dv}{dx}}{[v(x)]^2}$$

Chain Rule

$$\frac{d}{dx} (f(g(x))) = f'(g(x)) \times g'(x)$$

Leibnitz's theorem

$$\frac{d^n}{dx^n} (f \cdot g) = f^{(n)} \cdot g + n f^{(n-1)} \cdot g^{(1)} + \frac{n(n-1)}{2!} f^{(n-2)} \cdot g^{(2)} + \dots + \binom{n}{r} f^{(n-r)} \cdot g^{(r)} + \dots + f \cdot g^{(n)}$$

INTEGRALS

function

$$f(x) \frac{dg(x)}{dx}$$

$$x^n (n \neq -1)$$

$$\frac{1}{x}$$

$$e^x$$

$$\sin x$$

$$\cos x$$

$$\tan x$$

$$\operatorname{cosec} x$$

$$\sec x$$

$$\cot x$$

$$\frac{1}{a^2 + x^2}$$

$$\frac{1}{a^2 - x^2}$$

$$\frac{1}{x^2 - a^2}$$

$$\frac{1}{\sqrt{a^2 - x^2}}$$

$$\frac{1}{\sqrt{a^2 + x^2}}$$

$$\frac{1}{\sqrt{x^2 - a^2}}$$

$$\sinh x$$

$$\cosh x$$

$$\tanh x$$

$$\operatorname{cosech} x$$

$$\operatorname{sech} x$$

$$\operatorname{coth} x$$

integral

$$f(x)g(x) - \int \frac{df(x)}{dx} g(x) dx$$

$$\frac{x^{n+1}}{n+1}$$

$$\ell n|x|$$

$$e^x$$

$$-\cos x$$

$$\sin x$$

$$\ell n|\sec x|$$

$$-\ell n|\operatorname{cosec} x + \cot x| \quad \text{or} \quad \ell n\left|\tan \frac{x}{2}\right|$$

$$\ell n|\sec x + \tan x| = \ell n\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right|$$

$$\ell n|\sin x|$$

$$\frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$\frac{1}{2a} \ell n \frac{a+x}{a-x} \quad \text{or} \quad \frac{1}{a} \tanh^{-1} \frac{x}{a} \quad (|x| < a)$$

$$\frac{1}{2a} \ell n \frac{x-a}{x+a} \quad \text{or} \quad -\frac{1}{a} \operatorname{coth}^{-1} \frac{x}{a} \quad (|x| > a)$$

$$\sin^{-1} \frac{x}{a} \quad (a > |x|)$$

$$\sinh^{-1} \frac{x}{a} \quad \text{or} \quad \ell n(x + \sqrt{x^2 + a^2})$$

$$\cosh^{-1} \frac{x}{a} \quad \text{or} \quad \ell n|x + \sqrt{x^2 - a^2}| \quad (|x| > a)$$

$$\cosh x$$

$$\sinh x$$

$$\ell n \cosh x$$

$$-\ell n|\operatorname{cosech} x + \operatorname{coth} x| \quad \text{or} \quad \ell n\left|\tanh \frac{x}{2}\right|$$

$$2 \tan^{-1} e^x$$

$$\ell n|\sinh x|$$

Double integral

$$\iint f(x, y) dx dy = \iint g(r, s) J dr ds$$

where

$$J = \frac{\partial(x, y)}{\partial(r, s)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial s} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial s} \end{vmatrix}$$

SERIES

Powers of Natural Numbers

$$\sum_{k=1}^n k = \frac{1}{2} n(n+1); \quad \sum_{k=1}^n k^2 = \frac{1}{6} n(n+1)(2n+1); \quad \sum_{k=1}^n k^3 = \frac{1}{4} n^2(n+1)^2$$

Arithmetic
$$S_n = \sum_{k=0}^{n-1} (a + kd) = \frac{n}{2} \{2a + (n-1)d\}$$

Geometric (convergent for $-1 < r < 1$)

$$S_n = \sum_{k=0}^{n-1} ar^k = \frac{a(1-r^n)}{1-r}, \quad S_\infty = \frac{a}{1-r}$$

Binomial (convergent for $|x| < 1$)

$$(1+x)^n = 1 + nx + \binom{n}{2}x^2 + \dots + \binom{n}{r}x^r + \dots$$

where
$$\binom{n}{r} = \frac{n(n-1)(n-2)\dots(n-r+1)}{r!}$$

Maclaurin series

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \dots + \frac{x^k}{k!}f^{(k)}(0) + R_{k+1}$$

where
$$R_{k+1} = \frac{x^{k+1}}{(k+1)!}f^{(k+1)}(\theta x), \quad 0 < \theta < 1$$

Taylor series

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!}f''(a) + \dots + \frac{h^k}{k!}f^{(k)}(a) + R_{k+1}$$

where
$$R_{k+1} = \frac{h^{k+1}}{(k+1)!}f^{(k+1)}(a+\theta h), \quad 0 < \theta < 1.$$

OR

$$f(x) = f(x_0) + (x-x_0)f'(x_0) + \frac{(x-x_0)^2}{2!}f''(x_0) + \dots + \frac{(x-x_0)^k}{k!}f^{(k)}(x_0) + R_{k+1}$$

where
$$R_{k+1} = \frac{(x-x_0)^{k+1}}{(k+1)!}f^{(k+1)}(x_0+(x-x_0)\theta), \quad 0 < \theta < 1$$