Homework #11

Due on 11/23/2026, Monday, before class.

1. Exercise 3.4.16 on page 195

   3.4.16. Let the mutually independent random variables X_1, X_2, and X_3 be $N(0,1)$, $N(2,4)$, and $N(-1,1)$, respectively. Compute the probability that exactly two of these three variables are less than zero.

2. Exercise 3.4.29 on page 197

   3.4.29. Let X_1 and X_2 be independent with normal distributions $N(6,1)$ and $N(7,1)$, respectively. Find $P(X_1 > X_2)$.
   Hint: Write $P(X_1 > X_2) = P(X_1 - X_2 > 0)$ and determine the distribution of $X_1 - X_2$.

3. Exercise 3.5.2 (a) (b) on page 207 (you may use the Normal Distribution table on page 709)

   3.5.2. Let X and Y have a bivariate normal distribution with parameters $\mu_1 = 3$, $\mu_2 = 1$, $\sigma_1^2 = 16$, $\sigma_2^2 = 25$, and $\rho = \frac{3}{5}$. Using R, determine the following probabilities:

   (a) $P(3 < Y < 8)$.

   (b) $P(3 < Y < 8 | X = 7)$.

4. Exercise 3.5.9 on page 208

   3.5.9. Say the correlation coefficient between the heights of husbands and wives is 0.70 and the mean male height is 5 feet 10 inches with standard deviation 2 inches, and the mean female height is 5 feet 4 inches with standard deviation $1\frac{1}{2}$ inches. Assuming a bivariate normal distribution, what is the best guess of the height of a woman whose husband's height is 6 feet? Find a 95% prediction interval for her height.

5. Exercise 3.5.14 on page 209 (Graduates only)

3.5.14. Let $\mathbf{X} = (X_1, X_2, X_3)$ have a multivariate normal distribution with mean vector $\mathbf{0}$ and variance-covariance matrix

$$\mathbf{\Sigma} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}.$$

Find $P(X_1 > X_2 + X_3 + 2)$.
Hint: Find the vector $\mathbf{a}$ so that $\mathbf{aX} = X_1 - X_2 - X_3$ and make use of Theorem 3.5.2.

6. Exercise 3.6.2 on page 216 (you may use the t-Distribution table on page 710)

3.6.2. Let T have a t-distribution with 14 degrees of freedom. Determine b so that $P(-b < T < b) = 0.90$. Use either Table III or by using R.

7. Exercise 3.6.10 on page 217 (you may use the F-Distribution table starting on page 711)

3.6.10. Suppose F has an F-distribution with parameters $r_1 = 5$ and $r_2 = 10$. Using only 95th percentiles of F-distributions, find a and b so that $P(F \leq a) = 0.05$ and $P(F \leq b) = 0.95$, and, accordingly, $P(a < F < b) = 0.90$.
Hint: Write $P(F \leq a) = P(1/F \geq 1/a) = 1 - P(1/F \leq 1/a)$, and use the result of Exercise 3.6.9 and R.

For your reference, here's (**not part of the homework**):

3.6.9. Let F have an F-distribution with parameters r_1 and r_2. Argue that $1/F$ has an F-distribution with parameters r_2 and r_1.