Homework #9

Due on 11/02/2026, Monday, before class.

1. Exercise 2.6.1 (a) (b) (c) (f) on page 142

   2.6.1. Let X, Y, Z have joint pdf $f(x, y, z) = 2(x + y + z)/3,\ 0 < x < 1,\ 0 < y < 1,\ 0 < z < 1$, zero elsewhere.

   (a) Find the marginal probability density functions of $X, Y,$ and Z.

   (b) Compute $P(0 < X < \frac{1}{2}, 0 < Y < \frac{1}{2}, 0 < Z < \frac{1}{2})$ and $P(0 < X < \frac{1}{2}) = P(0 < Y < \frac{1}{2}) = P(0 < Z < \frac{1}{2})$.

   (c) Are X, Y, and Z independent?

   (f) Find the conditional distribution of X and Y, given $Z = z$, and evaluate $E(X + Y|z)$.

2. Exercise 2.6.4 on page 143

   2.6.4. A fair die is cast at random three independent times. Let the random variable X_i be equal to the number of spots that appear on the ith trial, $i = 1, 2, 3$. Let the random variable Y be equal to $\max(X_i)$. Find the cdf and the pmf of Y.
   Hint: $P(Y \leq y) = P(X_i \leq y,\ i = 1, 2, 3)$.

3. Exercise 2.7.7 on page 151 (Graduates only)

   2.7.7. Let X_1, X_2, X_3, X_4 have the joint pdf $f(x_1, x_2, x_3, x_4) = 24,\ 0 < x_1 < x_2 < x_3 < x_4 < 1$, 0 elsewhere. Find the joint pdf of $Y_1 = X_1/X_2,\ Y_2 = X_2/X_3, Y_3 = X_3/X_4, Y_4 = X_4$ and show that they are mutually independent.

4. Exercise 2.8.2 on page 153

   2.8.2. Let X_1, X_2, X_3, X_4 be four iid random variables having the same pdf $f(x) = 2x,\ 0 < x < 1$, zero elsewhere. Find the mean and variance of the sum Y of these four random variables.

5. Exercise 3.1.7 on page 163

3.1.7. Let the independent random variables X_1 and X_2 have binomial distribution with parameters $n_1 = 3$, $p = \frac{2}{3}$ and $n_2 = 4$, $p = \frac{1}{2}$, respectively. Compute $P(X_1 = X_2)$.
Hint: List the four mutually exclusive ways that $X_1 = X_2$ and compute the probability of each.

6. Exercise 3.1.27 on page 166 (Graduates only)

3.1.27. Let X have a geometric distribution. Show that

$$P(X \geq k + j \mid X \geq k) = P(X \geq j), \tag{3.1.9}$$

where k and j are nonnegative integers. Note that we sometimes say in this situation that X is **memoryless**.