Homework #3

Due on 09/16/2026, Wednesday, before class.

1. Exercise 6.1.2 on page 360

6.1.2. Let $X_1, X_2, \ldots, X_n$ represent a random sample from each of the distributions having the following pdfs:

(a) $f(x;\theta) = \theta x^{\theta-1}$, $0 < x < 1$, $0 < \theta < \infty$, zero elsewhere.

(b) $f(x;\theta) = e^{-(x-\theta)}$, $\theta \le x < \infty$, $-\infty < \theta < \infty$, zero elsewhere. Note that this is a nonregular case.

In each case find the mle $\hat{\theta}$ of θ.

2. Exercise 6.1.4 on page 361 (no need to show the consistency in (c))

6.1.4. Suppose $X_1, \ldots, X_n$ are iid with pdf $f(x;\theta) = 2x/\theta^2$, $0 < x \le \theta$, zero elsewhere. Note this is a nonregular case. Find:

(a) The mle $\hat{\theta}$ for θ.

(b) The constant c so that $E(c\hat{\theta}) = \theta$.

(c) The mle for the median of the distribution. Show that it is a consistent estimator.

3. Exercise 6.1.6 on page 361 (no need to show the consistency)

6.1.6. Suppose $X_1, X_2, \ldots, X_n$ are iid with pdf $f(x;\theta) = (1/\theta)e^{-x/\theta}$, $0 < x < \infty$, zero elsewhere. Find the mle of $P(X \le 2)$ and show that it is consistent.

4. Exercise 6.1.11 on page 362

6.1.11. Let $X_1, X_2, \ldots, X_n$ be a random sample from a $N(\theta, \sigma^2)$ distribution, where σ^2 is fixed but $-\infty < \theta < \infty$.

(a) Show that the mle of θ is $\overline{X}$.

(b) If θ is restricted by $0 \le \theta < \infty$, show that the mle of θ is $\widehat{\theta} = \max\{0, \overline{X}\}$.