0. Read Chapter 1: Introduction and Fundamentals.

1. Let random variable $X \sim \text{Binomial}\,(n, p)$, and denote $\lambda = np$. Show that the Poisson distribution is a limiting distribution of binomial distribution, i.e. as $n \to \infty$,

$$P\,(X = x) \to \frac{\lambda^x e^{-\lambda}}{x!},\; x = 0, 1, 2, \ldots$$

2. Derive the mean and variance for Poisson distribution using moment generating function.

3. Let p denote the probability that, for a particular tennis player, the first serve is good. Since $p = 0.4$, this player decided to take lessons in order to increase p. When lessons are completed, the hypothesis $H_0 : p = 0.4$ will be tested against $H_1 : p > 0.4$ based on 25 trials. Let Y denote the number of first serves that are good, and the critical region be defined by $C = \{y : y \geq 13\}$.

(a) Determine the significance level $\alpha = P\,(Y \geq 13 \mid p = 0.4)$.

(b) Find the power $1 - \beta$ at $p = 0.6$.

4. Let X be a discrete random variable taking on only positive integer values. Show that

$$E(X) = \sum_{i=1}^{\infty} P(X \geq i)$$