STAT 451: Computational Statistics, Spring 2020   Homework 1

Due date: January 24, 2020 (Friday), before class

1) Use simulation to approximate the following integrals. Describe your algorithm first. Attach your computer code and output.

   a) $\int_1^3 \frac{x}{(1+x^2)^2} dx$

   b) $\int_{-\infty}^{\infty} x^2 \exp\{-x^2\} dx$

   c) $\int_{-1}^{1} \int_{-1}^{1} |x - y| dx dy$

2) Consider the model given by $X \sim \text{lognormal}(0,1)$ and $\log Y = 9 + 3 \log X + \epsilon$, where $\epsilon \sim N(0,1)$ is independent of X. Use simulation to estimate $E[Y/X]$.

3) Use rejection sampling to estimate $E[X^2]$ when X has the density that is proportional to $q(x) = \exp\{-|x|^3/3\}$. Describe your algorithm first. Attach your R code and output. Be sure to count and report your acceptance ratio.