Stat 521 HW3 Fall 2012 Due September 21, 2012

Required Part:

0. Read §1c Eigenvalues and Reduction of Matrices.

1. Show that A^- is a g-inverse if and only if A^-A is idempotent and $R(A^-A) = R(A)$.

2. Let A be an $n \times n$ matrix with a in each diagonal position and b in each off-diagonal position.

(a) Show that $|A| = (a-b)^{n-1}\,[a + (n-1)b]$.

(b) Find the eigenvalues and eigenvectors of A.

(c) When does A have an inverse? Find A^{-1} when it exists.

3. Let G_1 and G_2 be n.n.d. matrices of the same order.

(a) Show that $G_1 + G_2$ is n.n.d. too.

(b) Show that $\mathcal{M}(G_1) \subset \mathcal{M}(G_1 + G_2)$.

4. Show that

(a) A symmetric matrix has a symmetric g-inverse.

(b) An n.n.d. matrix has an n.n.d. g-inverse.

Optional Part:

5. Let A be an $n \times n$ n.n.d. matrix and B be an $n \times n$ p.d. matrix. Show that there exists a nonsingular matrix N such that $NBN' = I$ and NAN' is diagonal with nonnegative diagonal entries.