Stat 521 HW7 **Fall 2012** **Due October 26, 2012**

Required Part:

0. Read §4b Tests of Hypotheses and Interval Estimation; §4c Problems of a Single Sample; §4d One-way Classified Data; and §4e Two-way Classified Data.

1. Consider the multiple regression model:

$$y_i = \beta_0 + \beta_1 x_{1i} + \cdots + \beta_{m-1} x_{m-1,i} + \epsilon_i,$$

$i = 1, \ldots, n$ with ϵ_i's iid from $N(0, \sigma^2)$.

(i) Give a sufficient condition on the x_{ji}'s under which $\beta_0, \beta_1, \ldots, \beta_{m-1}$ are all estimable.

(ii) If $\hat{\beta}_j$ denotes a least square estimate of β_j, then show that

$$\sum_{i=1}^{n} \left[y_i - \left(\hat{\beta}_0 + \hat{\beta}_1 x_{1i} + \cdots + \hat{\beta}_{m-1} x_{m-1,i} \right) \right] = 0.$$

2. Let $t_1, \ldots, t_k$ be unbiased estimators of a single parameter θ and $cov(t_i, t_j) = \sigma_{ij}$. Suppose $\Sigma = (\sigma_{ij})$ is positive definite. Find the linear function of $t_1, \ldots, t_k$ unbiased for θ and having minimum variance.

Hint: See (ii) of §1f.1 on page 60.

3. Show that if $t_1, \ldots, t_k$ are unbiased minimum variance estimators of the parameters $\theta_1, \ldots, \theta_k$, respectively, then $c_1 t_1 + \cdots + c_k t_k$ is the unbiased minimum variance estimator of $c_1 \theta_1 + \cdots + c_k \theta_k$.

Hint: Show that if T is an unbiased minimum variance estimator of θ, D is another estimator such that $E(D) = 0$, then $Cov(T, D) = 0$.