Stat 522 HW6 **Spring 2025** **Due March 5, 2025**

1. Suppose $X_1, \dots, X_n$ is a random sample from

$$\begin{pmatrix} Y \\ Z \end{pmatrix} \sim N_2\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix}\right)$$

Let r_{yz} denote the sample correlation coefficient of Y and Z computed from this random sample. Derive the distribution of

$$\frac{r_{yz}^2}{1 - r_{yz}^2}$$

(*Hint:* You may use the following result from Problem 5 of Hw4: The statistic $\frac{|S|}{|S_{11}| \cdot |S_{22}|}$ has the distribution of a product of $p-r$ independent beta-distributed random variables with parameters $(\frac{n-p+1}{2}, \frac{r}{2})$, $(\frac{n-p+2}{2}, \frac{r}{2})$, ..., $(\frac{n-r}{2}, \frac{r}{2})$, respectively.)

2. Suppose $S \sim W_p(n, \Sigma)$, the Wishart distribution with n degrees of freedom and $p \times p$ covariance matrix Σ. Derive the distribution of trace$(S\Sigma^{-1})$.

3. Justify (6-70) on page 317.
(*Hint:* You may follow the same procedure deriving Result 6.5 on page 309.)