Problem Set #1 - Math 569

1. (a) Show by using Reidemeister moves that $K_{3,2} \cong K_{2,3}$.

(b) Show by using Reidemeister moves that $E \cong E^*$.

(c) Show by using Reidemeister moves that $\overrightarrow{8_{17}} \cong \overrightarrow{8_{17}^*}$. (It turns out that $\overrightarrow{8_{17}} \neq \overrightarrow{8_{17}^*}$.)

(d) Prove that $(S^3, K_{n,m}) \cong (S^3, K_{m,n})$ for $\gcd(m,n) = 1$ (torus knots).
2. Think of $K_{m,n} \subset S^1 \times S^1$ as a curve embedded in the torus $S^1 \times S^1$. (We take $\gcd(m,n) = 1$). Show that there exists an orientation preserving surface homeomorphism $h: S^1 \times S^1 \to$ such that $h(S^1 \times S^1, K_{m,n}) = (S^1 \times S^1, \mathcal{U})$ where $(S^1 \times S^1, \mathcal{U}) = \begin{array}{c}
abla \in \text{trivial circle } \mathcal{U} \text{ on a torus.}
\end{array}

3. Compute a presentation for $\pi_1(E) \equiv \pi_1(S^3 - E)$, and use your result to prove that $E \not\cong T \otimes T = K_{2,3}$.

4. Prove that the Wirtinger presentation for a knot diagram $K$ gives a group $G$ that is invariant (up to isomorphism) under the Reidemeister moves.

5. $[M^3(K) \text{ is the famous Poincaré Manifold}]$

a) Compute a presentation for $\pi_1(M^3(K))$.

b) Show that $H_1(M^3(K)) \cong \text{trivial} \cong \{0\}$.