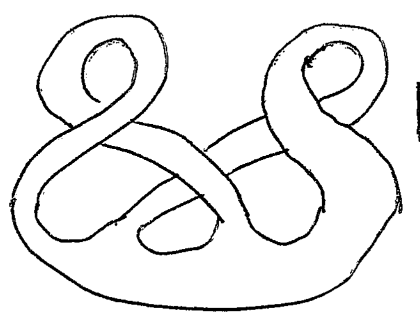
HW#3 – Math 569

1. Exercise on Poincaré Manifold Sheet.

2. Compute $\pi_1(M^3(K'))$ where

 K'

and prove that $M^3(K')$ is not homeomorphic to Poincaré manifold.

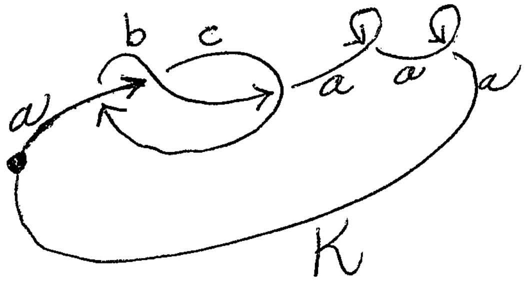
3.  F Show $\partial F \cong$ 

Find Seifert pairing, signature and Alex–Conway polynomial.

4. M^3() $\cong S^3$. Prove it.

5. Choose a knot different from trefoil or figure eight and workout everything you can about it.

Poincaré Manifold Via Surgery on +1 framed Trefoil



$\pi(K) = G = (a, b \mid aba = bab)$

$$\boxed{\begin{array}{l} c = b^{-1}ab \\ b = a^{-1}ca \\ a = c^{-1}bc \end{array}}$$

Longitude λ for this framing is

$\underline{\lambda = baca^{-2}}$ (obtained by starting from $\xrightarrow{a}$)

So $\lambda = ba(b^{-1}ab)a^{-2} = bab^{-1}aba^{-2}$.

$\therefore \pi_1(M^3(K)) \cong (a, b \mid aba = bab,\ bab^{-1}aba^{-2} = 1)$

$H_1(M^3(K)) = \pi_1(M^3(K))^{ab}$: $a^2b = b^2a \Rightarrow a = b$

$1 = bab^{-1}aba^{-2} = b$

$\Longrightarrow \underline{H_1(M^3(K)) \cong \{\phi\}}$.

$H = \pi_1(M^3(K)) \cong (a, b \mid aba = bab,\ a^2 = bab^{-1}ab)$

$a^2 = bab^{-1}ab$

$\Leftrightarrow a^3 = bab^{-1}aba = bab^{-1}bab = ba^2b$

So $H \cong (a, b \mid aba = bab,\ a^3 = ba^2b)$

Let $x = a$, $y = ab$. Then $yx = aba$

& $(ab)^3 = ababab = abaaba = (aba)^2 = (yx)^2$

$\phantom{(ab)^3 = ababab = {}} a(ba^2b)a = aa^3a = a^5$.

$\therefore x^5 = y^3 = (yx)^2$.

> $\underline{\text{Exercise}}$: Assuming only that $x^5 = y^3 = (yx)^2$, define $a = x$, $b = x^{-1}y$ and prove that
>
> 1. $aba = bab$
> 2. $a^3 = ba^2b$.
>
> Conclude that $\pi_1(M^3(K)) \cong (x, y \mid x^5 = y^3 = (yx)^2)$.