1. Exercise on Poincaré Manifold Sheet.

2. Compute $M^3(K')$ where

and prove that $M^3(K')$ is not homeomorphic to Poincaré manifold.

3. Show $\exists F \in \mathbb{Q}$

   Find Seifert pairing, signature and
   Alex-Conway polynomial.

4. $M^3(\infty) \cong S^3$. Prove it.

5. Choose a knot different from trefoil or figure eight and workout everything you can about it.
Poincaré Manifold Via Surgery on +1 framed Trefoil

\[ \pi_1(K) = \mathbb{Z} = (a, b | aba = bab) \]

\[
\begin{align*}
C &= b^{-1}ab \\
b &= a^{-1}ca \\
a &= c^{-1}bc
\end{align*}
\]

Longitude \( \lambda \) for this framing is

\[ \lambda = b a c a^{-2} \] (obtained by starting from \( a \rightarrow \))

So \( \lambda = ba(b^{-1}ab)a^{-2} = bab^{-1}aba^{-2} \).

\[ \therefore \pi_1(M^3(K)) \cong (a, b | aba = bab, bab^{-1}aba^{-2} = 1) \]

\[ H_1(M^3(K)) = \pi_1(M^3(K))^{ab} : \quad a^2 = b^{-1}a \quad \Rightarrow \quad a = b \]

\[ 1 = bab^{-1}aba^{-2} = b \]

\[ \Rightarrow \quad H_1(M^3(K)) \cong \mathbb{Z} \]

\[ H = \pi_1(M^3(K)) \cong (a, b | aba = bab, a^2 = bab^{-1}ab) \]

\[ \leftrightarrow a^3 = bab^{-1}aba \]

\[ \leftrightarrow a^3 = bab^{-1}aba = bab^{-1}bab = ba^2b \]

So \( H \cong (a, b | aba = bab, a^3 = ba^2b) \)

Let \( x = a, y = ab \). Then \( y x = aba \)

\[ (ab)^3 = ababab = ababa = (aba)^2 = (y x)^2 \]

\[ ab a^2 b = a a^3 a = a^5 \]

\[ \therefore \quad x^5 = y^3 = (yx)^2 \]

Exercise: Assuming only that \( x^5 = y^3 = (yx)^2 \)

define \( a = x, b = x^4 y, \) and prove that

1. \( aba = bab \)
2. \( a^3 = ba^2 b \)

Conclude that \( \pi_1(M^3(K)) \cong (x, y | x^5 = y^3 = (yx)^2) \).